Investment Dynamics and Financial Market Imperfections[1]

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October 22^{nd}, 2004

[1]We would like to thank Giuseppe Bertola, Roger Farmer, Fumio Hayashi, Thomas Hintermaier, Marco Pagano, Mirko Wiederholt and seminar participants at Madrid, Vienna, Florence and Salerno for their comments. All remaining errors are the author's.

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Abstract

This paper studies optimal firm investment in the presence of financial market imperfections. Financing frictions are modeled by letting the interest rate on the firm's debt depend on its capital structure. It is shown how the jointly endogenous investment and financing decisions are affected by the financial environment. Conditions are derived under which $Tobin's \; q$ remains a sufficient statistic for investment. We further derive a structural investment equation and a new test for the importance of financial market imperfections for firm-level investment. The empirical results imply that financial market imperfections do affect firm level-investment.

Keywords: Investment, Capital structure, $Tobin's \; q$

JEL classification: D92, E22

1 Introduction

Empirical implementation of the neoclassical theory of investment (see Tobin (1969), Lucas & Prescott (1971) and Hayashi (1982)) showed that $Tobin's$ q, measured as the market value of an average unit of capital, does not contain all the relevant information about investment, although under the standard assumptions of constant returns to scale and perfect capital markets it should be a sufficient statistic. Instead, coefficients on variables like current income, cash-flow or sales turn out to be estimated significantly different from zero when these variables are added to q-regressions (Hubbard (1997) and Caballero (1997) provide excellent surveys of results for firm level and aggregate investment data). A large empirical literature sparked by the work of Fazzari, Hubbard & Petersen (1988) (from now on simply FHP) attribute the empirical failure of the standard theory to imperfections in financial markets because they generally find that in firm-level data the departure from the theoretical predictions is most pronounced for firms which a priori are expected to be more severely financially constrained. Examples of firm characteristics which have been used to partition the sample include: dividend/income ratios (FHP), firm age and size (Gilchrist & Himmelberg (1995)), membership in a "keiretsu" (Hoshi, Kashyap & Scharfstein (1991)) and many others. While producing intriguing results this methodology has not been regarded as conclusive however and a quite critical assessment of this methodology can be found in Kaplan & Zingales (1997). More recently in fact, several studies have been published which relate the observed correlation between current cash flow and current investment after controlling for $Tobin's\ q$ to the presence of product market imperfections, learning effects or measurement error rather than financial market imperfections. Cooper & Ejarque (2000) present numerical simulations which reproduce the estimation results obtained by Gilchrist & Himmelberg (1995) in a model with perfect financial markets, but require a rather strong concavity of the production function. Alti (2003) introduces learning effects about the true productivity of the firm and manages to replicate the original FHP estimates. Erickson & Whited (2000) argue that biased estimates due to measurement error in q rather than financing frictions cause the empirical phenomenon.

In this paper, we tackle the question by explicitly introducing financial market imperfections into the q-model and allowing for a concave operating profit function[1]. We characterize the optimal investment and financial policy of the firm and derive an estimable investment equation, which takes into account potential discrepancies between *average* and *marginal q*. By proceeding in this structural way, we can gain some understanding of how the incentive to invest may differ from the market valuation of the installed capital and what other variables should be expected to help explaining investment. After all, the true incentive to invest, *marginal q*, adjusts endogenously to the existence of financial and real imperfections and therefore also carries information about the financing and production capabilities of

[1] The concavity of the operating profit function with respect to the capital stock could be due to both product market imperfections or decreasing returns to scale in production. We do not distinguish between those two sources because we focus on the question whether real or financial imperfections cause the Fazzari, Hubbard & Petersen (1988) results.

the firm. Some of this information will also be reflected in $Tobin's\ q$. We believe that this approach successfully addresses some of the concerns about the methodology used by FHP. First, our structural equation is valid for all firms and is not subject to the critique that sample splits are to a large degree arbitrary. Second, by allowing for both, real and financial imperfections, we address the concerns of Cooper & Ejarque (2000), who find that imperfect product markets or decreasing returns to scale in production can also explain the observed correlation between investment and current cash flow.

The next section introduces a deterministic model of optimal firm investment under imperfect financial markets. Section 3 characterizes the equilibrium and derives the conditions under which $Tobin's\ q$ should be a sufficient statistic for investment even if financial markets are imperfect. Section 4 derives a new test for the importance of financial market imperfections based on a structural investment equation and section 5 presents empirical results for a sample of Italian firms. We conclude by discussing the results and indicating open issues to be studied by future research.

2 Introducing financial variables into the q-model

Our way of introducing financial market imperfections is to allow the interest rate on the firm's debt to depend on the capital structure of the firm. In the microeconomic literature one can find many models of fundamental contractual problems in financial markets, which imply a relationship between the interest rate charged by the lender and the debt/capital ratio of the project to be financed (see Townsend (1979) or Gale & Hellwig (1985)). To keep the model tractable, we do not dig deeper into the microeconomic foundations and take the dependence of the interest rate on the capital structure as exogenously given[2]. Previously, Hayashi (1985) has provided an analysis of optimal investment with adjustment costs and imperfect financial markets, in which the cost of debt finance is increasing in the debt-capital ratio. In his model, different tax rates on dividends and capital gains also drive a wedge between the cost of retained earnings and the cost of new equity issue, and he distinguishes three different financing regimes of investment. He shows that under a specific assumption on bankruptcy costs, q-theory holds in two of the three regimes. However, he does not provide a general specification for an investment equation. The model we present below is simple enough to allow us to derive an estimable investment equation taking into account financial market and product market imperfections.

The model is set in a dynamic deterministic continuous time framework in order to focus on the interaction between the optimal investment and financial policy and the endogenous adjustment of *marginal* and $Tobin's\ q$.

[2] Another approach to contractual problems in credit markets is based on work by Hart & Moore (1994) and assumes imperfect enforceability of debt contracts. The equilibrium interest rate in this model is the safe interest rate r, but the size of the loan is restricted to be smaller than some multiple of the collateral supplied by the entrepreneur. In the appendix we explore the (similar) implications of this alternative way to model financial market imperfections.

Some papers explicitly differentiate between the financial market's valuation of the capital stock of the company and the manager's valuation of the firm's capital stock. We assume that those two measures of q are equal. The main differences to the standard q-model are the appearance of another state variable, $B(t)$ which denotes the stock of debt of the firm, and the statement of the objective function of the firm. In Appendix A.2 this formulation is derived from a model in which the objective of the firm is to maximize the market value of the firm's equity capital at time 0. The model can be written as

$$V(0) = \max_{\{I(t),X(t)\}_0^\infty} \int_0^\infty \Gamma(t) \begin{pmatrix} \Pi[z(t), K(t)] - \Psi[I(t), K(t)] \\ -\rho[r(t), B(t), K(t)]B(t) + X(t) \end{pmatrix} dt \tag{1}$$

subject to

$$\begin{aligned} \dot{K}(t) &= I(t) - \delta.K(t) \\ \dot{B}(t) &= X(t) \end{aligned}$$

where $\Gamma(t) = \exp \int_{\tau=0}^{t} -\gamma(\tau)\, d\tau$ and $\gamma(\tau)$ denotes the exogenous discount rate of the shareholders which control the financial and investment policy of the firm. $K(t)$ and $B(t)$ denote the size of the capital stock and debt of the firm, respectively. The capital accumulation equation is the usual one, with δ denoting the rate at which capital goods are assumed to decay. The debt accumulation equation simply defines $X(t)$ as the rate of change of debt at time t, implying that there are no costs of adjusting the capital structure. We also allow for an infinite rate of change in the state variables, so that at the initial point in time the firm could discretely adjust the stock of debt[3].

[3] Arrow & Kurz (1970) show that in a deterministic, concave, infinite horizon problem

The operating income function $\Pi [z(t), K(t)]$ captures any profits the firm generates from normal operations. We denote by $z(t)$ an exogenous driving variable that captures the state of business and is assumed to be known. The function $\Pi [.]$ is assumed to be twice continuously differentiable and concave in $K(t)$, $\Pi_K [.] > 0$, $\Pi_{KK} [.] \leq 0$. Operating income is zero, if the firm does not have any capital: $\Pi [z(t), 0] = 0$. $I(t)$ denotes the rate of gross investment. The standard adjustment cost function $\Psi [I(t), K(t)]$ is assumed to be twice continuously differentiable with respect to both arguments, strictly convex in $I(t)$, $\Psi_I [.] > 0$, $\Psi_{II} [.] > 0$ and $K(t)$,.$\Psi_K [.] < 0$, $\Psi_{KK} [.] \geq 0$. If investment is zero, adjustment costs are zero as well: $\Psi [0, K(t)] = 0$.

The next two terms in the objective function represent the non-standard part of the model. The cost of debt finance is $\rho [r(t), B(t), K(t)] B(t)$ which reflects the interest payments on the stock of debt $B(t)$. The interest rate on the firm's debt $\rho [.] > 0$ depends on the "riskless" interest rate $r(t)$ and the size of debt and capital of the firm. Since very low levels of debt are essentially riskless for the lenders, it is assumed that $\rho [r(t), 0, K(t)] = r(t)$. Further, the interest rate depends continuously on its arguments, with continuous second order partial derivatives and $\rho_B [.] \geq 0$, $\rho_K [.] \leq 0$ and $\rho_{KK} [.] \geq 0$. These assumptions imply that keeping other variables constant the interest rate increases with an increase in debt and decreases with an increase in the amount of capital. Importantly, we assume $\gamma(t) > r(t)$,

with continuous dynamics of the exogenous variables discrete adjustments (jumps) in state variables only accour in the initial period.

i.e. the discount rate of firm decision makers, $\gamma(t)$, is assumed to be bigger than the "riskless" rate, $r(t)$. The difference between these rates is an essential element of the analysis. If $r(t) = \gamma(t) = \rho[.]$ the model collapses to the standard perfect capital markets q-model. One way to justify this assumption is to appeal to the dual character of $\gamma(t)$ as discount rate and required rate of return. Then, by appealing to the "equity premium puzzle" which states that the spread between the rate of return on "risky" equity and the rate of return on "riskless" bonds is much higher than would be predicted by a standard representative agent capital asset pricing model one could justify this assumption. Our deterministic model is unable to capture a risk premium, but the difference between $r(t)$ and $\gamma(t)$ could be viewed as a reduced form of this effect.

From here, the analysis will proceed in two steps. First we characterize the solution of the model and present some numerical examples of optimal policies. Second, we derive the implications of the model for the specification of investment equations and clarify the relationship between *marginal* and $Tobin's\ q$ in the presence of financial market imperfections.

2.1 The optimal investment and financial policy

The necessary and sufficient FOC of the problem are given by:

$$\gamma(t) - \rho[r(t), K(t), B(t)] = \rho_B[r(t), K(t), B(t)]\, B(t) \tag{2}$$

$$q(t) = \Psi_I[I(t), K(t)] \tag{3}$$

$$\dot{q}(t) = (\gamma(t) + \delta)\, q(t) - (O_K[.] - \Psi_K[.] - \rho_K[.]\, B(t)) \tag{4}$$

the transversality condition

$$\lim_{t\to\infty} q(t) K(t) \Gamma(t) = 0$$

and the two dynamic constraints of the problem.

The state-space of the model is given by the stock of capital and *marginal* q, $\{K, q\}$. If the operating income function is strictly concave and the effect of the capital stock on adjustment costs and the interest rate is not too strong, a steady-state exists at which $q^* = 1$, and K^* and B^* are finite. If these conditions do not hold, it is still possible to determine the optimal investment rates and the optimal financial policy of the firm, but $q(t)$ does not converge to 1 and the firm invests according to the future path of $z(t)$, $\gamma(t)$ and $r(t)$. Importantly, the firm's stock of debt $B(t)$ is a jump variable. This is a result of assuming no adjustment costs of debt and frictionless access to equity finance, which reduces the dimension of the state space and equation (2) to a static relationship. The optimal amount of leverage is found at the point at which the marginal benefit of additional debt finance $\gamma(t) - \rho[r(t), K(t), B(t)]$ equals its marginal cost $\rho_B[r(t), K(t), B(t)] B(t)$. Since $\rho[r(t), K(t), 0] = r(t)$ and $\gamma(t) > r(t)$, the optimal amount of debt is positive at any time. We can use the implicit function theorem to see how $B(t)$ changes in response to changes in $K(t)$. We obtain the following expression for the derivative of $B(t)$ with respect to $K(t)$:

$$\frac{\partial B(t)}{\partial K(t)} = -\frac{\rho_K[.] + \rho_{BK}[.] B(t)}{2\rho_B[.] + \rho_{BB}[.] B(t)} \tag{5}$$

By using this expression, we can prove the following proposition:

Proposition 1 *If the discount rate $\gamma(t)$ and the „riskless" rate $r(t)$ are constant over time, the debt-capital ratio is also constant if the interest rate function is homogeneous of degree zero in $B(t)$ and $K(t)$.*

Proof. Homogeneity of degree zero implies: $-\rho_K[.]\,K(t) = \rho_B[.]\,B(t)$. Taking the derivative of this expression with respect to $B(t)$ yields $-\rho_{KB}[.]\,K(t) = \rho_{BB}[.]\,B(t) + \rho_B[.]$. Multiplying both sides of the equation by $B(t)$ and adding the resulting expression to the initial one we obtain

$$-\rho_K[.]\,K(t) - \rho_{KB}[.]\,K(t)\,B(t) = 2\rho_B[.]\,B(t) + \rho_{BB}[.]\,B(t)^2$$

By comparing the above equation with (5) we can see that it implies $\frac{\partial B(t)}{\partial K(t)} = \frac{B(t)}{K(t)}$, which is equivalent to the definition of a constant debt-capital ratio $\frac{\partial B(t)}{B(t)} = \frac{\partial K(t)}{K(t)}$. ■

This first result essentially means that if the interest rate only depends on the debt-capital ratio, optimal leverage does not depend on operating income, adjustment costs or the size of the firm, but only on the financial parameters $r(t)$, $\gamma(t)$, and $\rho[.]$. In general however, we expect optimal leverage to depend on fundamentals[4].

We see from (3) that the standard q-investment relationship continues to hold. Investment is positive if *marginal* q is bigger than 1 and negative if it is below 1. The pace of investment in the imperfect financial markets

[4] It would be a straightforward extension to let the interest rate on the firm's debt also depend explicitly on firm age (time t). Then optimal leverage would also change with the age of the firm.

case is still governed by the nature of adjustment costs. But what determines the incentive to invest? It is in the determination of *marginal* q, where the interaction between optimal financial and investment policy is most important. Integrating the dynamic equation for *marginal* q from 0 to ∞ assuming constant γ, we obtain:

$$q(0) = \int_0^\infty \exp\left[-(\gamma + \delta) t\right] \left(O_K[.] - \Psi_K[.] - \rho_K[.] B(t)\right) dt \tag{6}$$

This equation shows that the incentive to invest is unambiguously increased by the possibility of using debt. This follows from the assumptions for $\rho_K[.]$ and the result that optimal debt is always positive. The financial factor increasing the incentive to invest is the fact that holding the amount of debt constant, installing more capital decreases the cost of debt finance. The strength of this effect is captured by $\rho_K[.]$ and it is stronger the more debt the firm uses. At the same time, the optimal amount of debt depends on the size of the capital stock. Both quantities therefore endogenously adjust to each other and their optimal paths must be determined jointly.

It should also be pointed out that there is another interesting innovation in the expression for *marginal* q, which establishes a potentially important channel between the financial sector and the real sector of the economy. This is the fact that any change in the "riskless" interest rate $r(t)$ or the parameters of the relationship between debt, the capital stock and the interest rate affects $\rho_K[.]$ and therefore also directly the incentive to invest. The desired capital stock will adjust accordingly and there will be a significant effect on future investment rates. It is quite plausible, that this effect is one of the channels through which monetary policy and developments in the financial sector affect the real sector of the economy. In the next section we show some numerical examples which illustrate the way financial policy and the incentive to invest affect each other.

2.2 Some numerical examples

The numerical examples below show that imperfections in financial markets have considerable effects on investment. Figure I shows a plot of the debt-capital ratio paths of 3 firms characterized by the same decreasing returns to scale production function and linearly homogeneous adjustment technology, but facing different interest rate functions[5]. We also simulate the optimal investment policy of a firm operating in perfect capital markets. For this firm optimal financial policy is of course indeterminate. In our simulations all firms face the same cost of debt finance, $r = 0.04$, at a debt/capital ratio equal to 0. Also the initial capital stock, the operating income function and the adjustment cost function are equal in all 4 simulations. Any difference in investment behavior must therefore originate from the financing aspect of the problem.

A first result is that there is quite a large difference between the perfect financial markets case and the imperfect financial markets cases. The steady-

[5] The choices for the functional forms and parameters are the following: $\Pi[.] = 0.5K(t)^{0.5}$, $\Psi[.] = 0.5\frac{I(t)^2}{K(t)} + (1-0.1)I\left(t\right)$, $\gamma = 0.09, r = 0.04$ and the following three interest rate functions:

I: $\rho\left[.\right] = r + 0.04B\left(t\right)^{0.75}K\left(t\right)^{-1}$

II: $\rho\left[.\right] = r + 0.04B\left(t\right)^{1}K\left(t\right)^{-1}$

III: $\rho\left[.\right] = r + 0.08B\left(t\right)^{1.25}K\left(t\right)^{-1}$

state level of the capital stock in the former is 3.429, while in the latter case, the steady-state capital stocks lie in range from 2.088 to 2.427. Hence, investment is considerably lower if financial markets are imperfect. The structure of the financial market imperfection itself does not lead to such large differences among the firms. This is true, although the financial policies associated with each of these investment paths vary widely. The steady-state debt-capital ratios range from 0.539 to 0.858. Also, the paths of the debt/capital-ratio maybe increasing, decreasing or flat depending on the relative effects of the stocks of debt and capital on the interest rate. Figure 1 shows the results for the debt-capital ratio.

Figure 1 should be placed here.

The steady-state capital stock is increasing in the steady-state debt-capital ratio because firms adjust to a quickly increasing interest rate by choosing both a lower debt-capital ratio and a lower capital stock. Also the dynamics of the debt/capital ratio are governed by the shape of the interest rate function. Firm I ($\mu = 1$) chooses a constant debt/capital ratio because of the homogeneity of the interest rate function, its steady-state capital stock is 2.178 and its steady state debt-capital ratio is 0.625. Firm II ($\mu = 0.75$) chooses an increasing debt-capital ratio because for a constant debt/capital ratio, the interest rate decreases with a higher capital stock. Its steady-state debt-capital ratio is 0.858. The effectively lower user costs of capital make it optimal for the firm to invest at a higher rate and achieve a steady-state capital stock of 2.427, about 11% bigger than Firm I. Firm III ($\mu = 1.25$) instead chooses a decreasing path because for a constant debt-capital ratio, its interest rate increases with capital and selects a steady-state debt-capital ratio of 0.539. The firm therefore has a lower incentive to invest and its steady-state capital stock of 2.088 is about 14% below the capital stock of Firm II . After having illustrated how imperfections in financial markets affect the incentive to invest, we show in the next section how the existence of these imperfections affects the most widely used measure of this incentive, the market valuation of the firm's capital.

2.3 The relationship between $marginal$ and $Tobin's\ q$

Hayashi (1982) has shown that if the operating income function and the adjustment cost function are homogeneous of degree 1, then the incentive to invest, *marginal q*, equals the market valuation of the firm's capital, $Tobin's$ q. This result is the basis of most empirical work on investment, but it is unclear whether this result still holds in the presence of financial market imperfections. We cannot expect the result to hold in general but below we show that for two important special cases: if the interest rate on the firm's debt only depends on the debt-capital ratio and if collateral constraints are linear. We have shown above that even with imperfect financial markets, the incentive to invest is still captured by *marginal q*. In fact, equation (3) shows that *marginal q* is still a sufficient statistic for investment. We now show to what extent the true incentive to invest is reflected in $Tobin's\ q$.

$Tobin's\ q$, $q\,(0)$, is defined as the sum of market capitalization $V\,(0)$ and total debt $B(0)$ divided by the book value of capital $K(0)$

$$q(0) = \frac{V(0) + B(0)}{K(0)}$$

Unlike *marginal* q, *Tobin's* q is an observable quantity that can be used as an explanatory variable in empirical research. It is shown in Appendix A.3 that in the presence of financial market imperfections Hayashi's assumptions of constant returns to scale of the operating income and adjustment cost functions do not imply that *marginal* q equals *Tobin's* q. Instead, *marginal* q equals:

$$q(0) = \frac{V(0) + B(0)}{K(0)} + FLV \tag{7}$$

where

$$FLV = -\frac{1}{K(0)} \int_0^\infty \Gamma(t) \left\{ \left(\frac{\rho_B [.] B(t)}{\rho [.]} + \frac{\rho_K [.] K(t)}{\rho [.]} \right) \rho [.] B(t) \right\} dt \tag{8}$$

Expression (8) shows that *Tobin's* q needs to be adjusted by a part of the market value derived from the use of debt in the future to arrive at the true incentive to invest. We refer to this adjustment term as the "Future Leverage Value", FLV. When we are willing to assume that the interest rate function $\rho[.]$ is of the constant elasticity form, then we can use (8) to determine the sign of the correction term and we see that correcting for financial market imperfections will decrease the value of *Tobin's* q, if and only if

$$-\frac{\rho_K [.] K(t)}{\rho [.]} < \frac{\rho_B [.] B(t)}{\rho [.]}$$

or equivalently that the negative of the elasticity of the interest-rate function with respect to capital is lower than the elasticity with respect to debt. In fact, we can prove the following proposition:

Proposition 2 *Firms operating in imperfect financial markets with profit and adjustment cost functions which are homogeneous of degree 1 exhibit marginal q smaller than* $Tobin's\ q$*, if the elasticity of the interest-rate with respect to capital is lower in absolute value than the elasticity with respect to debt for all values of* $B(t)$ *and* $K(t)$.

Proof. The sign of FLV is determined by an integral over sums of future elasticities of the interest rate function. If $-\frac{\rho_K[.]K(t)}{\rho[.]} < \frac{\rho_B[.]B(t)}{\rho[.]}$ holds for all $B(t)$ and $K(t)$, then all elements of the integral are positive and the integral itself is positive as well. Positivity of the integral implies that *marginal* q is smaller than $Tobin's\ q$. ■

The intuition for this result is that as the firm accumulates capital, it will also increase the stock of its debt. If the interest rate reacts more strongly to the increase in debt than to the increase in the capital stock the average financing cost of capital will increase if investment is financed in the same way as the existing capital stock. The financial value of the average unit of capital will therefore decrease over time. It is this change in the average financing cost of capital which decreases the incentive to invest relative to the market value of the average unit of capital. If the interest rate only depends on the debt/capital-ratio, optimal leverage is constant and no correction is necessary:

Proposition 3 *If the interest rate function* $\rho[r(t), K(t), B(t)]$ *is homogeneous of degree zero in* $K(t)$ *and* $B(t)$*, then* $FLV = 0$ *and if also the profit function and the adjustment cost function are homogeneous of degree 1,* $Tobin's\ q$ *is equivalent to marginal* q.

Proof. See Appendix A.4. ■

In Appendix A.1 we show that a similar result holds for a model in which financial market imperfections are modeled by collateral constraints. In the usual case of a linear collateral constraint of the form $B(t) \leq \alpha K(t)$, $marginal\ q$ still equals $Tobin's\ q$ under Hayashi's homogeneity assumptions. If the collateral constraint is nonlinear however, an adjustment of $Tobin's\ q$ is necessary to capture the true incentive to invest. The next section will use the results obtained here to derive an estimable investment equation which explicitly takes into account the possibility that $marginal\ q$ is different from $Tobin's\ q$. It is shown that under some specific assumptions we can identify the source of the wedge between the incentive to invest and the market valuation of capital by estimating a structural equation for the investment rate.

3 Implications for empirical work

The results above imply that without some guidance from a structural model it is difficult to detect empirically whether financial market imperfections are important determinants of investment. In particular, the procedure used by FHP is not well-suited to test for the importance of financial market imperfections. Cooper & Ejarque (2000) illustrate the argument that adding a current cash-flow or operating income variable to the q-equation cannot be the basis for a test for financial market imperfections because a strictly concave operating income function also yields cash-flow effects whose magnitude varies across subsamples even if financial markets are perfect FHP and their followers also failed to show that optimal investment plans under imperfect financial markets actually would produce cash-flow effects theoretically. We showed above that even if financial market imperfections were important determinants of investment at the firm level, the empirical researcher who simply adds current cash-flow or current operating income to the regression might wrongly conclude that they do not matter, if he bases his test on the FHP procedure. This is because, as we have shown above, the market value of equity not only carries information about the operating profitability of the capital stock, but also about the cost of financing the capital. The market value therefore endogenously adjusts to the financing frictions that some firms may face. In some cases, e.g. when collateral constraints are linear or when the interest rate depends only on the debt-capital ratio, the adjustment is such that it fully captures the effects of financing frictions on investment. One then observes that the standard $Tobin's$ q-investment relationship continues to hold, but as we showed above, this does not imply that financial factors do not matter for investment. In order to meaningfully test for the importance of financial market imperfections for firm investment we must also take into account the financial policy of the firm and the possibility that the operating income function is strictly concave. We now derive a structural investment equation that fulfills these requirements.

We start by making some assumptions on functional forms and on firm heterogeneity. We will from now on assume that the adjustment cost function is common for all firms and takes a simple quadratic form homogeneous of degree 1 in its arguments with $\theta > 0$. Moving to discrete time it is defined for firm i in period t as

$$\Psi\left[I_{it}, K_{it}\right] = I_{it} + \frac{\theta}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it}$$

The first-order condition for investment is linear in this case and can be written as

$$\left(\frac{I}{K}\right)_{it} = \left(\frac{\delta\theta - 1}{\theta}\right) + \frac{1}{\theta} q_{it} \tag{9}$$

We also assume that the operating income and interest rate functions are simple power functions

$$\begin{aligned} O[K_{it}, z_{it}] &= z_{it} K_{it}^{\alpha} \\ \rho\left[K_{it}, B_{it}\right] &= r_t + \eta\left(B_{it}^{\beta+\lambda} K_{it}^{-\beta}\right) \end{aligned}$$

with $0 < \alpha \leq 1$ and $\eta, \lambda > 0$ again assuming that the parameters are common for all firms.

Observe that with $\alpha = 1$ and $\lambda = 0$, the model behaves like the standard q-model. The homogeneity assumptions are satisfied and $Tobin's\ q$ is a sufficient statistic for investment. The debt-capital ratio of the firm is constant. A departure of any of the two parameters from this value will break the relationship and while *marginal* q still remains a sufficient statistic, $Tobin's$ q must be adjusted in order to arrive at the true incentive to invest. It is shown in Appendix A.4 that the relation between *marginal* and $Tobin's\ q$ in the present case takes the following form

$$q\left(0\right) = \frac{V\left(0\right) + B\left(0\right)}{K\left(0\right)} + PDV_{\alpha} + FLV \tag{10}$$

where PDV_α is the correction due to the fact that $\alpha < 1$ or that the operating income function is strictly concave. Similarly, FLV reflects the correction term due to the non-homogeneity of the interest rate function. Analytical expressions can be derived for each of these corrections necessary. Assuming a fixed discount rate γ, they take the following form

$$PDV_\alpha = -\frac{1}{K(0)} \int_0^\infty \exp[-\gamma t] \{(1-\alpha) z(t) K(t)^\alpha\} dt \qquad (11)$$

$$FLV = -\frac{1}{K(0)} \int_0^\infty \exp[-\gamma t] \left\{ \lambda\eta \left(\frac{B(t)}{K(t)} \right)^\beta B(t)^\lambda \right\} dt \qquad (12)$$

The sign of these correction terms only depend on α and λ. $PDV_\alpha < 0$, if $0 < \alpha < 1$ because a strictly concave profit function implies that the marginal unit of capital earns less than the previously installed units. Hence, $Tobin's\ q$ needs to be corrected downwards in order to arrive at the true incentive to invest. Similarly, $FLV < 0$ if $\lambda > 0$ where the argument here rests on the fact that with $\lambda > 0$ financing large capital stocks is costlier on average than financing small capital stocks for any given debt-capital ratio. The cost of debt finance is more sensitive the amount of debt the firm issues than with respect to the capital stock it owns. Therefore the financial gain from financing capital with debt is largest for the first units of capital installed. Hence, $Tobin's\ q$ must be corrected downward to arrive at the true incentive to invest. The inverse argument holds, if $\lambda < 0$.

Despite having derived a relationship between the observable variable $Tobin's\ q$ and the investment rate, we do not have an estimable equation yet. The reason for this is that the correction terms derived above involve future variables and are unobservable just like $marginal\ q$. To use these expressions in empirical work, it is therefore necessary to approximate them. In Appendix A.5 we derive linear approximations to the correction terms which can be written in discrete time as

$$\begin{aligned} P\tilde{D}V_{\alpha} &= \alpha_1 + \alpha_2 K_{it} \\ F\hat{L}V &= \lambda_1 + \lambda_2 \left(\frac{B}{K}\right)_{it} + \lambda_3 B_{it} \end{aligned}$$

where the α_i and λ_i are defined in equations 33 and 34. Substituting these approximations into (9) yields

$$\left(\frac{I}{K}\right)_{it} = \left(\frac{\delta\theta - 1 + \alpha_1 + \lambda_1}{\theta}\right) + \frac{1}{\theta} q_{it} + \frac{\alpha_2}{\theta} K_{it} + \frac{\lambda_2}{\theta}\left(\frac{B}{K}\right)_{it} + \frac{\lambda_3}{\theta} B_{it} + \varepsilon_{it} \quad (13)$$

where $\varepsilon_{it} = \tau_i + \upsilon_{it}$ is an error term with a random firm-specific and time-invariant component τ_i and an i.i.d. component $\upsilon_{it} \sim N(0, \sigma)$. We assume that the τ_i, the υ_{it} and the regressors are independent of each other.

If the profit function is strictly concave, we should observe a positive coefficient[6] on the scale variable K, since $\frac{\alpha_2}{\theta} > 0$ if $0 < \alpha < 1$. Analogously, if the interest rate function is homogeneous, we should not observe significant coefficients λ_2 and λ_3 because no correction for $Tobin's\ q$ is needed. For $\lambda > 0$, the interest rate is more sensitive with respect to the amount of debt than with respect to the capital stock and average financing costs are increased if future capital units are financed with the same debt-capital

[6] This result might be surprising, since the true incentive to invest is lower than $Tobin's$ q in the case of a concave profit function, but becomes clearer if one recognizes that the we use a linear approximation here and while the total correction to $Tobin's\ q$ is negative for all values of K, the correction term becomes smaller in absolute size the bigger the installed capital stock is because the discrepancy between average and marginal profitability is decreasing in the capital stock for the simple power function we have assumed.

ratio. To reflect this effect, $Tobin's\ q$ must be adjusted downward and hence $\lambda_2 < 0$. The reverse argument holds, if $\lambda < 0$, in this case, $\lambda_2 > 0$, and $Tobin's\ q$ of course must be adjusted upward. In addition to the leverage ratio, also the scale of debt affects the incentive to invest if the interest rate function is non-homogeneous. λ_3 is positive only if $0 < \lambda < 1$. If $\lambda < 0$ or $\lambda > 1$, we expect a negative λ_3[7]. If $\lambda = 1$ the correction is proportional to the debt-capital ratio and we obtain $\lambda_3 = 0$ although $\lambda_2 < 0$. Figure 2 illustrates our results in this section. For a given α, it shows optimal financing paths for different values of λ.

Figure 2 should be placed here.

In the next section we apply equation 13 to test for the importance of financial market imperfections and strict concavity of the operating income function.

4 Empirical evidence

The data we use for empirical work is constructed from an 11-year balanced panel of balance sheet and income statement data for 87 listed Italian companies with a total of 870 observations[8] from the Worldscope financial

[7]With $\lambda < 0$ the reasoning is analogous to the one for the concavity of the profit function. The total correction to $Tobin's\ q$ must be positive, but it is at the same time decreasing in the debt level because the per unit cost savings are declining with an increase in the level of debt.

[8]We lose one year of observation for each company because we are interested in beginning of period stocks and investment in a given period, but companies report end-of-period stocks simultaneously with investment in a given period.

database. The dataset is fairly small for microeconometric standards but yields comparable information on most items of the balance sheet and income statement, including gross capital expenditures, fixed capital, long and short-term debt, book value of equity, market capitalization,... The sum of long-term and short-term debt at the beginning of the period corresponds to our definition of the variable B. We take capital expenditures to correspond to our I-variable and the beginning of period net value of property, plant and equipment to correspond to K. Unfortunately, for some firms and years information on capital expenditures or market capitalization is missing and we therefore exclude these observations (136 observations lost). Also, some of the companies contained in the dataset are holding companies with consolidated balance sheets only for some years[9]. We exclude the observations of such companies for the years in which their balance sheets are unconsolidated. Further, companies that have made large acquisitions within the period covered are excluded as well. At the end, we are left with an unbalanced sample of 649 observations on 78 companies with the number of observations ranging from 1 to 10.

Running an FHP-style augmented q-regression[10] on the full dataset and two subsamples split according to the size of the capital stock[11] and using

[9] These companies are Ifil, Fin. Part. spa, Schiapparelli spa, Finmeccanica spa, Pirelli&C. sapa, Camfin spa, Simint spa, Parmalat Finanziaria spa, Bulgari spa, Mediaset spa, Istituto Finaziario Industriale spa, Cofide spa.

[10] We include year and sector dummies to capture general business cycle and sector-specific effects and use a standard random-effects model for estimation.

[11] The cutoff level is to a large extent arbitrary and we have chosen a value of approx. 35 mil Euros for the capital stock. We have experimented with different thresholds and

the operating income rate as the state of business indicator variable we find results similar to the ones in the literature. $Tobin's\ q$ is highly significant in all three samples but the operating income rate is significant only in the small firm and the overall sample. Further the point estimate of the coefficient on the operating income rate is biggest for the sample of small firms and decreases as the sample contains less small firms. This suggests that as shown in the previous literature $Tobin's\ q$ is not a sufficient statistic for investment, at least for the subset of small firms. However, as we showed before these findings do not imply that financial market imperfections affect firm investment because theoretically the significant coefficient on the operating income rate could be due to two separate effects: strict concavity of the operating income function or non-homogeneity of the interest rate function..

In order to find out whether financial market imperfections affect firm investment, we need a different procedure. Our model of optimal firm behavior outlined above suggests that we should apply equation (13) to the full dataset to make progress on this issue. We initially estimate a random effects specification in levels to obtain preliminary estimates under the assumption that there is no correlation between the error term ε_{it} and the regressor variables. Table 1 summarizes the results who generally lend support for our structural procedure and for the importance of financial market imperfections for investment.

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other cutoff criteria (number of employees, level of sales) and results were qualitatively similar.

Table 1 should be placed here.

When the q-regression is augmented by a cash flow variable and the regressors implied by our structural model the coefficient on cash-flow is no longer significant. This is shown in column 1 of Table 1. But the correlation with the investment rate is not picked up by the capital stock variable but by the debt-capital ratio. To the extent that our approximations of PDV_α and FLV are valid, this suggests financial factors drive a wedge between $Tobin's$ q and marginal q. The fact that the coefficient on the debt-capital ratio is negative and the coefficient on the stock of debt is insignificant suggests that $\lambda = 1$ is the empirically most relevant parametrization of the theoretical model. The results from a fixed effects specification not reported here are qualitatively similar and we also report the Breusch-Pagan and Hausman specification tests which both support the random effects specification. We test for serial correlation of the residuals by regressing residuals on their lagged values and report the p-value of the joint coefficient.

Hayashi & Inoue (1991) and Erickson & Whited (2000) suggest that the error term in the random effects specification employed could be correlated with the right hand side regressors due to endogeneity of regressors or measurement error. They suggest to use GMM-estimation on first-differenced variables in order to correct for potential endogeneity biases. Both papers find that after correcting for endogeneity and measurement biases cash-flow no longer affects firm-level investment. Table 1 reports the results of the GMM-estimation of equation (13) in first differences.

Table 2 should be placed here.

We estimate the equation via a GMM-procedure which allows for general heteroscedasticity and serial correlation in the estimated residuals. We choose the current and lagged levels of the regressors as instruments in the regression of first-differences. The p-values reported are derived from Huber-White heteroscedasticity-consistent standard errors. We also report the outcomes of the standard J-Test for overidentifying restrictions and an Wald-test for joint significance of parameters. The estimation results illustrate the robustness of our results to endogeneity bias. In the regression of first-differences, the coefficient on $Tobin's\ q$ is again highly significant and somewhat higher than in the levels regression. The signs of all coefficients except for the highly insignificant stock of debt are the same across the two specifications. While the coefficient on the debt-capital ratio is almost unchanged, the coefficient on the capital stock becomes significant after accounting for endogeneity bias which suggests that both real and financial market imperfections drive a wedge between $Tobin's\ q$ and the incentive to invest.

Apart from the implications for investment equations our theory also carries implications for the financial choices of the firms. In a perfect capital markets world, the financial policy of the firm is undetermined and we therefore expect that firm's choices are widely dispersed. In principle any debt-capital combination could be optimal in such a setting and a cross-plot of the logarithm of debt against the logarithm of capital should not have a particular structure. A priori we would expect a "cloud" of points.

Our imperfect capital markets model instead predicts a particular shape for this plot. Given our power function assumption and the empirical results above, the model implies that the points are dispersed along a straight line with a positive slope that is smaller than 1 since for the case of $\lambda > 0$, the debt-capital ratio should be decreasing with the capital stock of the firm.

Figure 3 should be placed here.

Figure 3 shows that the data are in line with the predictions of the model and the empirical results from the investment regression. Firms with more capital do tend to have lower debt-capital ratios because the interest rate reacts more strongly to an increase in debt than to an increase in capital. A simple OLS regression of log capital on log debt reveals that the slope of the regression line is significantly lower than 1, although only slightly so, suggesting a rather value for the exponent of the debt-capital ratio in the interest rate function β.

5 Conclusions

Firms operating in imperfect financial markets simultaneously choose both an optimal financial and an optimal investment policy. We have shown above that there is a direct relationship between the firm's capital structure, the firm's investment policy, and the extent of the financing friction. Firms for which access to financial markets is difficult, face interest rates that increase quickly with the amount of debt finance the firms use. Such firms will find it optimal to have relatively low initial levels of debt and capital and will mostly rely on equity to finance their investments. Their future leverage value is positive however and as these firms grow they increasingly rely on corporate debt to finance investments. Firms with easy access to financial markets will tend to have higher capital stocks and use more leverage from the beginning. But their future leverage value is negative and their debt-capital ratio is decreasing in the capital stock. If the interest rate only depends on the debt/capital ratio, the optimal debt/capital ratio is constant. If also the operating income and adjustment cost functions are homogeneous of degree 1, then $Tobin's\ q$ equals $marginal\ q$ even if financial markets are imperfect.

Our results imply that in order to determine whether financial market imperfections are important, one needs to analyze jointly the investment and financial decisions of the firms. The empirical results suggest that imperfections in financial markets do affect firm investment. We find evidence that the debt-capital ratio decreases with the size of the firm suggesting that the elasticity of the interest rate is greater with respect to debt than with respect to capital. The non-homogeneity of the interest rate function also breaks the equivalence between $average$ and $marginal\ q$ and seems to cause the correlation between investment and current operating income that is documented by previous empirical work. We do not find evidence that the adjustment cost function or the operating income function are non-homogeneous.

It would be important to find out in future research whether these results can be confirmed for other countries and other datasets. A possible limitation of our analysis is that we have not considered taxes and transaction costs in equity markets. These issues were considered by Hayashi (1985) and he finds that optimal policies are much more complex in this case. Other issues we have not considered is irreversibility of investment and the possibility of default. To tackle these questions a much less tractable stochastic model would be required. Whether any of these issues are important for the study of firm investment must at this point be answered by future research.

A Appendix

A.1 Collateral constraints

Another well known model of financing frictions due to Hart & Moore (1994) finds that the firm will face a collateral constraint of the form:

$$B\left(t\right) \leq \theta K\left(t\right) \tag{14}$$

This model is based on symmetric information, but the inability of creditors to punish the defaulting entrepreneur stronger than by taking away his wealth, makes lenders hesitant to lend more than some fraction of his current net worth. The interest rate charged to the firm will be the safe rate of interest $r\left(t\right)$. In the present case, the creditor is a firm, its wealth is the capital it owns and we assume that $0 < \theta < 1$. Using this approach the model reads:

$$\max_{\{I(t),X(t)\}_0^\infty} \left[\int_0^\infty \Gamma\left(t\right)\left(\Pi\left[.\right] - \Psi\left[.\right] - r\left(t\right)B\left(t\right) + X\left(t\right)\right)dt\right]$$

subject to

$$\begin{aligned}
\dot{K}(t) &= (I(t) - \delta.K(t))\,dt \\
\dot{B}(t) &= X(t)\,dt \\
B(t) &\leq \theta K(t)^{\sigma}
\end{aligned}$$

Facing such a constraint, the firm would still invest according to the q-rule, but again the value of q would be somewhat different from the perfect capital markets case. In fact, it can be shown that if the interest rate on debt is lower than the discount rate, the firm would always choose to operate at the collateral constraint, decreasing its cost of capital as much as possible. We then have

$$B(t) = \theta K(t)^{\sigma}$$

and the deterministic first-order optimality conditions are

$$\begin{aligned}
\dot{K}(t) &= (I(t) - \delta.K(t))\,dt \\
\dot{q}(t) &= (\gamma(t) + \delta)\,q(t) - \left(\Pi_K[.] - \Psi_K[.] + (\gamma(t) - r(t))\,\sigma\theta K(t)^{\sigma-1}\right)
\end{aligned}$$

and the transversality condition is

$$\lim_{t\to\infty} q(t)\,K(t)\,\Gamma(t) = 0$$

With constant γ, *marginal* q is given by

$$q(0) = E_t\left[\int_0^{\infty} \exp\left[-(\gamma+\delta)\,t\right]\left(\Pi_K[.] + \sigma\theta(\gamma - r(t))\,K(t)^{\sigma-1} - \Psi_K[.]\right)dt\right] \tag{15}$$

Clearly, the higher are θ and σ, the higher is also the shadow value of capital $q(t)$ and consequently the higher is investment. The standard investment-

marginal q relationship still holds

$$q(t) = \Psi_I [I(t), K(t)]$$

Current cash-flow or operating income still do not affect investment because the marginal financing cost to the firm would not change, if cash-flow or profit were higher! It would still be equal to the cost of additional equity finance $\gamma(t)$. The optimal investment and financial policies still must be determined jointly however.

Marginal and *Tobin's* q are equivalent the collateral constraint is linear $(\sigma = 1)$ and the homogeneity assumptions apply.

Proposition 4 *If the operating income and adjustment cost functions are homogeneous of degree 1 and the collateral constraint is linear, Tobin's* q *equals marginal* q

$$q(0) = \frac{V(0) + B(0)}{K(0)} \tag{16}$$

Proof. If $\sigma = 1$ we obtain from (29) defined in Appendix A.4

$$\frac{d}{dt} \{[q(t) K(t) - B(t)] \Gamma(t)\} \tag{17}$$

$$= [-\Pi_K [.] K + \Psi_K [.] K + \theta (\gamma(t) - r(t)) K + \Psi_I [.] I - X + \gamma(t) B] \Gamma(t) \tag{18}$$

after applying the homogeneity assumptions and adding and subtracting $r(t).B(t)$ inside the brackets we get

$$= -[\Pi[.] - \Psi[.] - r(t) B(t) + X(t)] \Gamma(t) + \tag{19}$$

$$[(\gamma(t) - r(t)) B(t) - \theta (\gamma(t) - r(t)) K(t)] \Gamma(t) \tag{20}$$

Now realizing that the collateral constraint $B(t) = \theta K(t)$ is always binding along an optimal path and integrating from 0 to ∞ and using the transversality condition we obtain Hayashi's result:

$$q(0) = \frac{V(0) + B(0)}{K(0)}$$

■

The intuition for this result is based on the fact that the firm in this model always operates at the collateral constraint and the constraint is being relaxed proportionally to the increase in the capital stock. Given homogeneity, the two terms in (8) now exactly offset each other and the market value of the average unit of capital employed by the firm therefore exactly equals the shadow value of the marginal unit of capital.

A.2 Deriving the objective function

The optimal investment problem of the firm is stated as maximizing the market value of the equity of the firm

$$V(0) = \max_{\{I(t), X(t)\}_0^\infty} p^S(0) N(0) \tag{21}$$

subject to

the capital and debt accumulation equations

$$\begin{aligned} dK(t) &= [I(t) - \delta.K(t)] \, dt & (22) \\ dB(t) &= X(t) \, dt & (23) \end{aligned}$$

the flow of funds constraint

$$D(t) = \Pi[.] - \rho[r(t), K(t), B(t)] B(t) - \Psi[.] + X(t) + p^S(t) \dot{N}(t) \tag{24}$$

and the pricing equation for the shares of the firm

$$\gamma(t) = \frac{\dot{p}^S(t)}{p^S(t)} + \frac{D(t)}{p^S(t).N(t)} \tag{25}$$

Here $D(t)$ is the dividend rate, $N(t)$ is the number of stocks issued at time t and $p^S(t)$ is the market price of a share at time t.

Taking the time derivative of $V(t) = p^S(t)\,N(t)$, we obtain

$$\dot{V}(t) = \dot{p}^S(t).N(t) + p^S(t).\dot{N}(t) \tag{26}$$

Multiplying (25) by $p^S(t).N(t)$ we obtain

$$\dot{p}^S(t)\,N(t) = \gamma(t)\,V(t) - D(t) \tag{27}$$

Substituting this expression into (26) and substituting from (24), we get

$$\dot{V}(t) = \gamma(t)\,V(t) - \Pi[.] + \Psi[.] + \rho[.]\,B(t) - X(t) \tag{28}$$

Solving this differential equation with random coefficients in $V(t)$, for starting time equal to 0 and horizon s to arrive at

$$\begin{aligned} V_0^s &= C.\exp\left[\int_0^s \gamma(\tau)\,d\tau\right] + \\ &\int_0^s \exp\left[\int_0^t \gamma(\tau)\,d\tau\right](\Pi[.] - \Psi[.] - \rho[.]\,B(t) + X(t))\,dt \end{aligned}$$

so that maximizing $V(0)$ over an infinite horizon is equivalent to maximizing

$$= \max_{\{I(t),X(t)\}_0^\infty} \int_0^\infty \Gamma(t)\,(\Pi[.] - \Psi[.] - \rho[.]\,B(t) + X(t))\,dt$$

A.3 The relation between *marginal* and *Tobin's q*

From (29) in Appendix A.4 we know that

$$\begin{aligned}&\frac{d}{dt}\left\{\left[q\left(t\right)K\left(t\right)-B\left(t\right)\right]\Gamma\left(t\right)\right\}\\=\quad&\left[-\Pi_{K}\left[.\right]K+\Psi_{K}\left[.\right]K+\rho_{K}\left[.\right]BK+\Psi_{I}\left[.\right]I-X+\gamma B\right]\Gamma\left(t\right)\end{aligned}$$

Adding and subtracting $\Pi\left[.\right]-\Psi\left[.\right]-\rho\left[.\right]B\left(t\right)$ to the right hand side of this equation, we obtain after integrating from 0 to ∞

$$\begin{aligned}q\left(0\right)K\left(0\right)\quad=\quad&V\left(0\right)+B\left(0\right)-\int_{0}^{\infty}\Gamma\left(t\right)\left(\Pi\left[.\right]-\Pi_{K}\left[.\right]K\right)dt\\&-\int_{0}^{\infty}\Gamma\left(t\right)\left(\Psi_{K}\left[.\right]K+\Psi_{I}\left[.\right]I-\Psi\left[.\right]\right)dt\\&-\int_{0}^{\infty}\Gamma\left(t\right)\left[\left(\rho_{B}\left[.\right]B+\rho_{K}\left[.\right]K\right)B\right]dt\end{aligned}$$

Using the specific functional forms we assume, this confirms our expression (10).

A.4 Hayashi's result for the baseline model

We start from the observation that the following equation holds along any optimal path:

$$\begin{aligned}&\frac{d}{dt}\left\{\left[q\left(t\right)K\left(t\right)-B\left(t\right)\right]\Gamma\left(t\right)\right\}\\=\quad&\left[\dot{q}\left(t\right)K\left(t\right)+q\left(t\right)\dot{K}\left(t\right)-\dot{B}\left(t\right)-\gamma\left(t\right)q\left(t\right)K\left(t\right)+\gamma\left(t\right)B\left(t\right)\right]\Gamma\left(t\right)\end{aligned}\tag{29}$$

where $\Gamma\left(t\right)=\exp\left(\int_{0}^{t}-\gamma\left(\tau\right)d\tau\right)$. Then substituting from the dynamic equation for $q\left(t\right)$, (4), the capital accumulation equation and the optimality condition for $I\left(t\right)$, (3),and dropping time indices we obtain

$$=\left[-\Pi_{K}\left[.\right]K+\Psi_{K}\left[.\right]K+\rho_{K}\left[.\right]BK+\Psi_{I}\left[.\right]I-X+\gamma B\right]\Gamma\left(t\right)\tag{30}$$

after cancelling offsetting terms. Now by applying the homogeneity assumptions $\Pi_K[.]\,K = \Pi[.]$ and $\Psi_K[.]\,K + \Psi_I[.]\,I = \Psi[.]$ and adding and subtracting $\rho[.]\,B(t)$ inside the brackets we can write

$$= \quad -\left[\Pi[.] - \Psi[.] - \rho[.]\,B(t) + X(t)\right]\Gamma(t) \tag{31}$$

$$+\left[(\gamma(t) - \rho[.])\,B(t) + \rho_K[.]\,B(t)\,K(t)\right]\Gamma(t) \tag{32}$$

which after integrating from 0 to ∞ and using the transversality condition yields

$$q(0)\,K(0) - B(0) = V(0) - FLV$$

which directly implies (7). If in addition $\rho[.]$ is homogeneous of degree 0 then $\rho_K[.]\,K + \rho_B[.]\,B = 0$ and using optimality condition (2) we obtain

$$FLV = -\frac{1}{K(0)}\int_0^\infty \Gamma(t)\left\{(\rho_B[.]\,B(t) + \rho_K[.]\,K(t))B(t)\right\}dt = 0$$

A.5 Approximating the correction terms

For $\alpha = 1$ the correction term in 11 equals 0. If $0 < \alpha < 1$ the capital stock of the firm converges towards a steady state assuming constant z and we can approximate the correction term by evaluating it at the steady state to obtain

$$PDV_\alpha = -\frac{(1-\alpha)\,z}{\gamma}K^{*(\alpha-1)}$$

and after taking a linear approximation we get

$$P\hat{D}V_\alpha = -\left(\frac{(1-\alpha)\,zK^*}{\gamma} + \frac{(1-\alpha)^2\,z}{\gamma}\right)K^{*(\alpha-2)} + \frac{(1-\alpha)^2\,zK^{*(\alpha-2)}}{\gamma}K_{it} \tag{33}$$

Similarly, for $\lambda = 0$ the correction term in 12 equals 0 and the debt/capital ratio is constant. We approximate the second correction term by evaluating it under the assumption of a constant debt/capital ratio and a constant debt-growth rate $b < \gamma$ which yields

$$FLV = \frac{\lambda\eta}{(b-\gamma)}\left(\frac{B(0)}{K(0)}\right)^{\beta+1} B(0)^{\lambda-1}$$

and by taking a linear approximation around the steady state amounts of debt and capital yields

$$F\hat{L}V = \left(\lambda^2 - \beta\frac{B^*}{K^*}\right)\xi + \lambda(\beta+1)\xi\left(\frac{B}{K}\right)_{it} + \frac{(\lambda^2-\lambda)\xi}{B^*}B_{it} \qquad (34)$$

where $\xi = \left(\eta B^{*\beta+\lambda}K^{*-(\beta+1)}\right)/(b-\gamma) < 0$.

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B Tables and Figures

Table 1: Testing for financial market imperfections

Dependent variable: $(I/K)_{it}$ Explanatory variables		
q_{it}	0.035* (0.004)	0.033* (0.004)
$(\Pi/K)_{it}$	-	0.022 (0.024)
$(B/K)_{it}$	-0.049* (0.011)	-0.046* (0.011)
K_{it}	-0.022 (0.056)	-0.021 (0.055)
B_{it}	0.005 (0.069)	0.003 (0.069)
Constant	0.172* (0.064)	0.177* (0.063)
Breusch-Pagan[a]	125.26	116.10
Serial correlation[b]	0.403	0.366
Hausman[c]	18.55	31.17

Note: a set of time and industry dummies was included; estimates are standard random-effects estimates; we report coefficients with their standard errors in parentheses; stars denote coefficients which are significant at the 5%-level

[a] Test-statistic of the Breusch-Pagan LM-test for random effects specification distributed as Chi-squared(1)

[b] P-value of the joint coefficient on the lagged residuals in a regression of residuals on their lagged values

[c] Test-statistic of the Hausman misspecification test distributed as a Chi-squared(13)

Table 2: Taking into account endogeneity

Dependent variable: $D(I/K)_{it}$ Explanatory variables	
DQ_{it}	0.051* (0.013)
$D(B/K)_{it}$	-0.057* (0.022)
DK_{it}	-0.317* (0.111)
DB_{it}	-0.001 (0.092)
Constant	0.052 (0.058)
Wald[a]	29.82
J-statistic[b]	2.262

Note: a set of time and industry dummies was included; estimates are second stage GMM-estimates using current and lagged levels as instruments; we report coefficients with robust standard errors in parentheses; stars denote coefficients which are significant at the 5%-level

[a] Wald is the statistic of a Wald-test on the joint significance of the coefficients distributed as a Chi-squared(4)

[b] Test-statistic of Hansen's J-test for overidentifying restrictions distributed as a Chi-squared(4)

Figure 1:

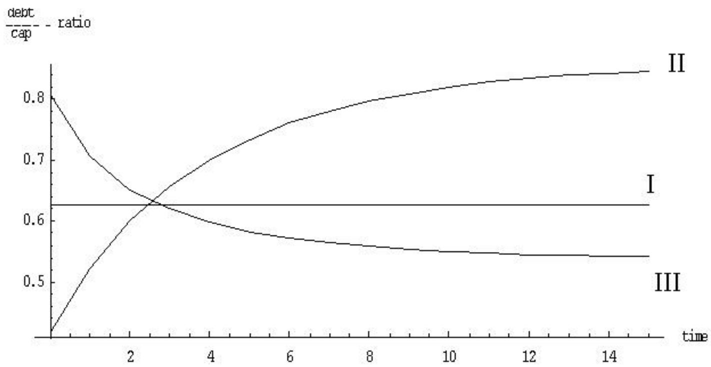

Figure 1: The dynamics of the debt-capital ratio

Figure 2:

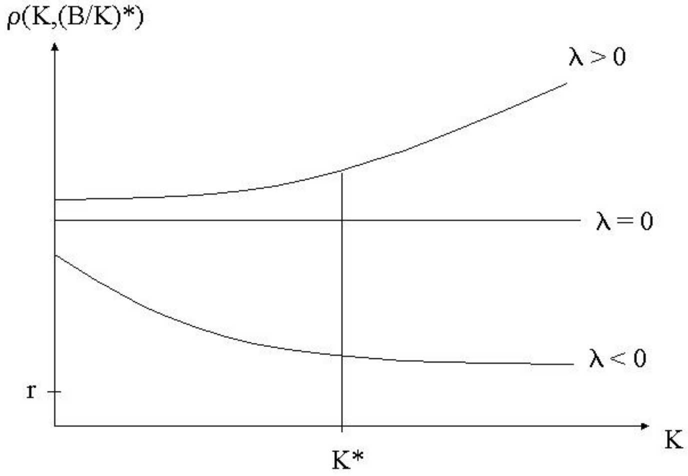

Figure 2: Optimal financing costs and the capital stock

Figure 3:

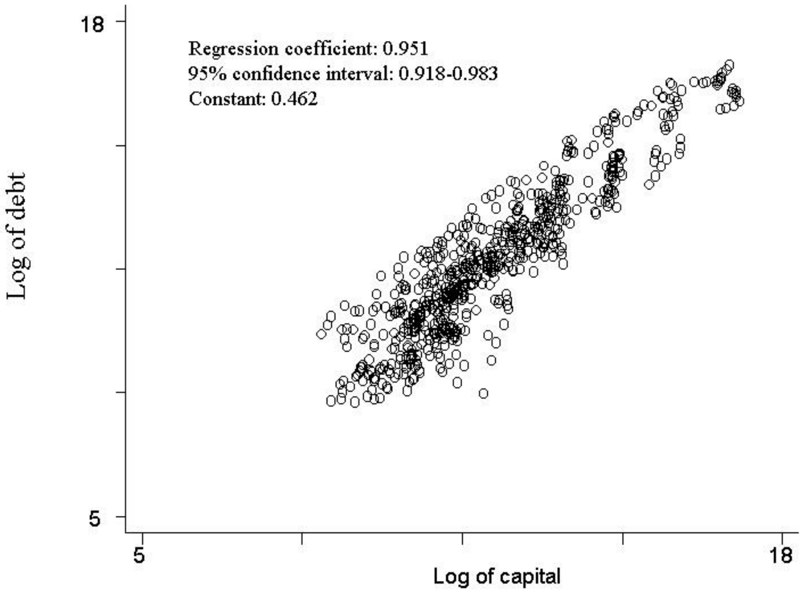

Figure 3: Stocks of debt and capital for Italian firms