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Journal of Financial Intermediation 13 (2004) 96–131

Journal of Financial Intermediation

www.elsevier.com/locate/jfi

Value-at-risk vs. building block regulation in banking ☆

Thomas Dangl [a,*] and Alfred Lehar [b,1]

[a] *Department of Managerial Economics and Industrial Organization, Vienna University of Technology, Theresianumgasse 27, A-1040 Vienna, Austria*

[b] *Department of Business Studies, University of Vienna, Brünner Straße 72, A-1210 Vienna, Austria*

Received 1 January 2002

Abstract

Existing regulatory capital requirements are often criticized for only being loosely linked to the economic risk of the banks' assets. In view of the attempts of international regulators to introduce more risk sensitive capital requirements, we theoretically examine the effect of specific regulatory capital requirements on the risk-taking behavior of banks. More precisely, we develop a continuous time framework where the banks' choice of asset risk is endogenously determined. We compare regulation based on the Basel I building block approach to value-at-risk or 'internal model'-based capital requirements with respect to risk taking behavior, deposit insurance liability, and shareholder value. The main findings are: (i) value-at-risk-based capital regulation creates a stronger incentive to reduce asset risk when banks are solvent, (ii) solvent banks that reduce their asset risk reduce the current value of the deposit insurance liability significantly, (iii) under value-at-risk regulation the risk reduction behavior of banks is less sensitive to changes in their investment opportunity set, and (iv) banks' equityholders can benefit from risk-based capital requirements.

JEL classification: G21; G28; G12

Keywords: Banking regulation; Basel Accord; Risk management

☆ A previous version of this paper was entitled "Basel Accord vs. value-at-risk regulation in banking."

* Corresponding author.

E-mail addresses: thomas.dangl@tuwien.ac.at (T. Dangl), alfred.lehar@univie.ac.at (A. Lehar).

1 Part of this work was done while Lehar was at the University of British Columbia, Canada.

1042-9573/$ – see front matter
doi:10.1016/j.jfi.2003.06.001

1. Introduction

> [W]e have no choice but to continue to plan for a successor to the simple risk-weighting approach to capital requirements embodied within the current regulatory standard. While it is unclear at present exactly what that successor might be, it seems clear that adding more and more layers of arbitrary regulation would be counter productive. We should, rather, look for ways to harness market tools and market-like incentives wherever possible, by using banks' own policies, behaviors, and technologies in improving the supervisory process.
>
> Greenspan (1998)

The impact of bank regulation on risk-taking behavior has been a major focus during periods of severe financial crises, such as the 1999 Asian experience. While there is still an ongoing debate whether regulation is beneficial at all,[2] the regulatory framework continues to evolve over time as a number of regulatory guidelines have been issued by the Basel Committee on Banking Supervision and by national regulators.

One of the milestones in banking regulation is the 1988 Basel Accord[3] (also called Basel I), where regulators establish minimum capital requirements for banks. The idea is to mandate banks to hold capital as a safety cushion in order to ensure bank solvency. Banks holding riskier assets must hold more capital as they have a higher probability of failure. To link the required capital to the riskiness of a banks' assets, the accord assigns assets to different risk buckets,[4] and specifies bucket-specific equity requirements (risk weights). Whereas capital requirements are homogeneous within each of these buckets, the economic risk of assets assigned to the same risk bucket may vary substantially (e.g., all corporate loans have to be backed by 8% of capital regardless of the companies' ratings).[5] This fact gives rise to criticism of the Basel I Accord since it opens the opportunity for 'regulatory capital arbitrage' by 'intra-bucket' risk shifting, i.e., increasing the risk of the bank's assets without increasing the capital requirements. For this reason, several regulatory agencies have proposed linking minimum capital requirements to economic risk more closely.[6]

Regulators have recognized this problem and there have been two important steps towards enhanced risk sensitiveness of capital requirements since the release of Basel I. An amendment to the Basel I Accord[7] incorporates the market risk of the trading book into the international banking regulation framework. It offers banks the opportunity to compute minimum capital requirements for proprietary trading activities using a value-at-risk

[2] See e.g. Freixas and Rochet (1997, p. 257) for a survey.

[3] See Basel Committee on Banking Supervision (1988).

[4] All assets are assigned to one of four buckets. These buckets coarsely classify the riskiness of the respective contract, e.g., loans to OECD governments, loans to OECD banks and other OECD public sector entities, residential mortgage loans, loans to the private sector. For a more detailed description see, e.g., Jorion (2000).

[5] Dimson and Marsh (1995) analyze the relationship between economic risk and capital requirements for different regulatory frameworks using trading book positions of UK securities firms. They find that the building block approach leads only to modest correlation between capital requirements and total risk.

[6] See Santos (2000) or Meyer (1998) who notes, for example: "[C]apital arbitrage also undermines the effectiveness of our capital rules and creates some economic distortions."

[7] See Basel Committee on Banking Supervision (1996a).

approach. Recently, the Basel Committee released the second proposal for the New Basel Capital Accord[8] (also called Basel II). The newly proposed Internal Ratings Based Approach, while still a bucket building method, shows greater risk sensitiveness due to a finer granularity of the risk buckets and a dynamic assignment of loans to buckets based on the internal rating of the loan contracts.[9]

These changes in capital regulation are intended both for obtaining more precise measurements of risk and to create the appropriate risk-taking incentives for banks. The question then is: how do the different regulatory capital requirements affect the risk-taking behavior of banks? The aim of this paper is to theoretically examine this question. We set up a continuous time framework allowing banks to choose between two different asset portfolios that are characterized by different levels of risk. We study the optimal risk-taking behavior of banks when capital requirements have different risk sensitivity. Specifically we compare a simple Basel I building block (BB) approach and a value-at-risk (VaR) based approach as two genuine examples, recognizing that current regulations including Basel II lie between these polar cases. We also examine the effect on equity value and on the fair up-front deposit insurance premium, and derive policy implications for prudent bank regulation.

We find that there is room for a Pareto improvement by switching from BB regulation to VaR regulation in the sense that equityholders of well-capitalized banks, in addition to the deposit insurance corporation, gain from adopting the new regulatory environment. Carefully adjusted, the VaR-based regulation provides the proper incentive for well-capitalized banks to reduce asset risk by rewarding low-risk banks with lower capital requirements. When responding to this incentive (i.e., reducing their portfolio's risk), banks significantly lower the value of the deposit insurance liability. This behavior also increases the value of the bank charter (a sound investment policy increases the expected lifetime of the bank), and as such, equityholders prefer this new regulation. However, VaR regulation does not generally dominate BB-based capital requirements. Applying excessively high panic factors—as one possible example—may lead to inefficient early closure, and thus, reduce the bank's equity value compared to the BB framework. We show that neither the BB nor the VaR approach generally prevents banks from switching to the high-risk portfolio when they are in financial distress. Finally, we point out that adjusting the VaR approach implies carefully harmonizing the auditing intensity and regulatory capital requirements with each other in order to provide the proper risk-reduction incentive. In a comparative static analysis, we solve for the minimum level of auditing that regulators have to perform to induce risk reduction. We find that under VaR-based regulation, less auditing has to be performed and that the corresponding audit intensity is less sensitive to changes in the banks' investment opportunity set. Thus, our findings support the Basel Committee's recognition

[8] See Basel Committee on Banking Supervision (2001), according to the recent press release (see Basel Committee on Banking Supervision, 2002), the final version is to be published in 2003, with implementation planned at the end of 2006.

[9] In their analysis of the Basel II Accord, Altman and Saunders (2001) and Linnell (2001) criticize that the granularity of the buckets still remains to coarse and propose risk weights that "will bring regulatory capital closer to economic capital estimates."

of capital requirements and auditing policy as equally important pillars of the new capital accord.

There are two branches of literature related to our approach. The first addresses the issue of bank regulation in a continuous time framework. Merton (1977) derives the insurance premium of a fixed-length deposit contract applying the Black and Scholes (1973) option pricing framework. Merton (1978) introduces random audits by the regulator and derives the fair up-front price of deposit insurance under the assumption of a constant volatility of the bank's assets. Pennacchi (1987) considers risk-taking incentives by banks, where he defines risk in terms of financial leverage. He also points out the importance of regulatory response to a bank failure and compares direct payments to depositors to merging a failed bank. Fries et al. (1997) consider optimal bank closure rules balancing social bankruptcy costs against future auditing costs. They find incentives for managers to take risk, where risk is defined as the volatility of the underlying state variable and not as leverage, and they derive subsidy policies and equity support schemes that eliminate these risk taking incentives by linearizing the equityholders' value function. Finally, Bhattacharya et al. (2002) derive optimal closure rules that eliminate risk-taking incentives for managers, at least in the region where the bank is adequately capitalized. All these models assume that the volatility of the underlying state variable is constant. The existence of a risk-taking incentive is deduced solely from the convexity of the equityholders' value function. However, the process of risk shifting is not explicitly considered.

The second branch of the literature examines risk shifting in a continuous-time corporate finance setting. Ericsson (1997) and Leland (1998) introduce models where equityholders are allowed to switch from one risk level to another. Their goal is to price corporate securities and to derive the optimal capital structure policy of firms in the presence of agency costs arising from the asset substitution opportunity. While the modeling technique of these papers is similar to our approach, the economic context in banking is substantially different. Due to deposit insurance, debt can be raised at the riskless rate. Consequently, a conflict of interest evolves between equityholders and the deposit insurer. To prevent the exploitation of the insurance system, banks have to satisfy regulatory constraints which are enforced by an auditing mechanism. Our paper explores the incentives of these regulatory rules on risk taking as well as the optimal auditing policy.

The paper is organized as follows. Section 2 describes the model. Section 3 derives the general solution for claims on the banks assets. Section 4 compares BB and VaR regulation and explores the risk-taking incentives created by these mechanisms. Section 5 derives comparative statistics, considers welfare effects and gives some policy implications for prudent regulation, and in Section 6 we conclude.

2. Model

As in Merton (1974), the value of the banks assets V is assumed to follow a geometric Brownian motion. However, we extend this framework by allowing the bank's management to choose between two asset portfolios with different risk. More precisely, there is a 'low-risk' portfolio available whose dynamics are geometric Brownian with volatility σ_L and drift $\mu(\sigma_L)$ as well as a 'high-risk' portfolio, characterized by σ_H and $\mu(\sigma_H)$, with

$\sigma_H > \sigma_L$. At any instant in time management[10] has the freedom to substitute the current asset portfolio with the alternative portfolio, thereby changing the risk level of the underlying assets. Thus, our model explicitly allows for asset substitution. We assume that this substitution is costly in that a certain small fraction k of the asset value V is lost at any switch between portfolios. The bank's portfolio is assumed to include a major proportion of loans and other assets for which a shift in the risk structure is opaque for regulators. As the regulatory agency has no information on the bank's investment choice, it has to perform audits to learn the portfolio's risk.

To keep the model feasible, the portfolio choice is restricted to a discrete choice, i.e., the bank is either fully invested in the low-risk portfolio or in the high-risk portfolio. Formally, the asset value process of the bank can be written as

$$\mathrm{d}V = \begin{cases} (\mu(\sigma_L) - \delta_L)V\,\mathrm{d}t + \sigma_L V\,\mathrm{d}z_L & \text{bank owns the low-risk portfolio,} \\ (\mu(\sigma_H) - \delta_H)V\,\mathrm{d}t + \sigma_H V\,\mathrm{d}z_H & \text{bank owns the high-risk portfolio,} \\ -kV & \text{on asset substitution,} \end{cases}$$
$$V(0) = V_0 > 0, \tag{1}$$

where $\mu(\sigma_L)$ and $\mu(\sigma_H)$ are the total expected returns on the asset value V of the low-risk and of the high-risk portfolio, respectively. The differentials $\mathrm{d}z_L$ and $\mathrm{d}z_H$ are the increments of (possibly correlated) standard Wiener processes representing the random shocks the two portfolio values are exposed to. Since a combination of the two portfolios is not permitted, correlation has no effect on the choice, so the distinction between $\mathrm{d}z_L$ and $\mathrm{d}z_H$ is suppressed in the remainder of the paper. The instantaneous variance of the process V is $\sigma_L^2 V^2$ and $\sigma_H^2 V^2$ depending on the current risk level. Hence, the state of the bank is characterized by its location in the two dimensional state space $[0, \infty) \times \{\sigma_L, \sigma_H\}$ over the ranges of V and σ.

We assume that the bank has issued deposits with face value c/r (where r is the riskless rate of interest) requiring a continuous coupon flow c. These deposits are fully insured, so that in case of bankruptcy the depositors receive the full face value of their deposits. Equityholders have limited liability and are the residual claimholders of the bank's assets. If the asset value V is not sufficiently high to cover the claim of the depositors upon closure of the bank, the difference is borne by the deposit insurance corporation.

The holder of the assets earns a profit flow which is a certain proportion $\delta \in \{\delta_L, \delta_H\}$ of the portfolio value V. In addition to this cash flow, banks are able to generate an extra profit flow $\pi \in \{\pi_L, \pi_H\}$. This flow originates from special screening (see Allen, 1990 and Ramakrishnan and Thakor, 1984) and monitoring abilities of banks (see the Bhattacharya and Thakor (1993) review), and possibly from bank services such as liquidity provision (see Kashyap et al., 2002; Diamond, 1997; and von Thadden, 1999) and access to the payment system. Alternatively, we may interpret this extra profit as rents from imperfect competi-

[10] In line with most of the previous literature, the management's interests are assumed to be perfectly aligned with the equityholders'. A recent contribution by John et al. (2000) explicitly considered the agency conflict between equityholders and management and examined the interesting idea of linking bank regulation to management compensation.

tion, barriers to entry, exclusive access to cheap deposits, or tax benefits. The prospect of this future gains creates a charter value for the equityholders of the bank.[11]

Equityholders, as the residual claimants, are responsible to maintain the obligations of the bank. Whenever the profit flow from holding the asset portfolio, $\delta V + \pi$, is less than the required interest payment c, the equityholders have the choice to either inject money to guarantee solvency in order to keep the prospect of future benefits from running the bank, or alternatively, they may voluntarily close the bank. Thus, we focus on the bank's optimal investment decision, i.e., the optimal choice of the risk level and the closure level (see the discussion of the bank's strategy bellow).

Apart from voluntary closure, there is the possibility of forced closure by the regulatory authorities if the bank is not in accordance with the regulatory mechanism implemented. We consider regulatory mechanisms $\langle \lambda, B(\sigma)\rangle$ characterized by:

(i) an auditing intensity λ and
(ii) by a closure threshold $B(\sigma)$.

Specifically:

- Audits are assumed to occur randomly following a Poisson process with intensity λ. That means, we model an audit counter A defined by the stochastic differential equation

$$\mathrm{d}A = \begin{cases} 1 & \text{with probability } \lambda\,\mathrm{d}t, \\ 0 & \text{with probability } 1-\lambda\,\mathrm{d}t, \end{cases} \qquad A(0) = 0, \tag{2}$$

  which is incremented by one at any occurrence of an audit.
- The closure threshold $B(\sigma)$ determines the consequences of an audit by partitioning the state space of the bank into a 'closure region' ($V < B(\sigma)$) and a 'continuation region' ($V \geqslant B(\sigma)$). When an audit occurs and the bank's state is found to not be in accordance with regulatory requirements, the bank is forced to close. Due to the fact that our model allows for only two levels of asset risk (σ_L and σ_H), only the two critical thresholds $B(\sigma_L)$ and $B(\sigma_H)$ are relevant for the bank.

For a given regulatory mechanism $\langle \lambda, B(\sigma)\rangle$, bank management sets an optimal response in order to maximize equity value. At any state the available choices are:

(i) *stick* to the current risk level,
(ii) *switch* the level of asset risk, or
(iii) *close* the bank.

[11] The excess cash flow depends on the choice of the asset portfolio and is assumed to be constant as long as the bank sticks to its current portfolio. An alternative framework to model a charter value used by Decamps et al. (2003) is an incomplete market setting where banks are able to generate excess asset growth.

In particular, a strategy **S** is a mapping from the state space into the space of available choices,

$$\mathbf{S}:(V,\sigma)\rightarrow\{\textit{stick},\textit{switch},\textit{close}\}.$$

In technical terms, switching and closure points are absorbing barriers to the asset value process. While the first hit of a closure point results in the default of the bank, the first hit of a switching point $(\widehat{V},\sigma_H)$ absorbs the high-volatility process and creates a low-volatility process at $((1-k)\widehat{V},\sigma_L)$, i.e., switching from the high-risk asset portfolio to the low-risk portfolio destroys a fraction k of the asset value due to trading costs. Analogously, a switching point at $(\widehat{V},\sigma_L)$ absorbs the low-volatility process and creates one with high volatility at $((1-k)\widehat{V},\sigma_H)$. The decision to *stick* means to leave the current risk level unchanged.

Obviously, the possible structure of such a strategy could be very complex. However, from previous work on controlling Brownian motion we know that so-called *control limits policies* are optimal when there are lump-sum costs associated with the control effort.[12] This means that there exist regions where it is optimal to leave the system without control effort and to intervene only if the state of the bank hits certain upper or lower limits. Therefore, we study the class of strategies **S** where switching points and closure points are boundaries of intervals with constant volatility, i.e., where for given volatility the partition of the state space with **S** = *stick* is the union of open intervals. Inside these intervals of stable volatility the asset value V follows a simple (uncontrolled) geometric Brownian motion, see (1). Consequently, given a strategy **S** the value of any claim contingent on the bank's asset value can be obtained by standard contingent claims analysis when proper boundary conditions are applied at the respective switching and closure points (see Section 3).

Concluding this section, we will summarize the different claims contingent on the state of the bank (V,σ) that will be used to analyze the model and give their characteristics.

- The *market value of deposits*—denoted as $D(V,\sigma)$—is the market value of the non-insured coupon flow provided by the bank. In contrast to the insured contract held by depositors, which is always worth c/r, the claim D is exposed to default risk. Furthermore, the loss in asset value caused by the management's asset substitution strategy is regarded when evaluating D, i.e., the holders of D implicitly bear a certain proportion of the switching costs.
- The *value of the deposit insurance* is denoted as $DI(V,\sigma)$. This is the current value of possible future expenditures necessary to guarantee the full face value to depositors in case of bank closure. Obviously, the value of the deposit insurance is the difference between the insured value of deposits and the market value of the coupon flow. Thus,

$$DI(V,\sigma)=\frac{c}{r}-D(V,\sigma). \tag{3}$$

[12] See Harrison et al. (1988) and Taksar et al. (1988) for a mathematically rigorous treatment and Dixit (1991) or Dixit (1993) for the economic intuition behind the valuation and optimality conditions. The costs that arise on asset substitution are the lump-sum costs that make *control limits policies* optimal in our case.

- The *charter value*, denoted by $CV(V,\sigma)$, is the current value of the excess profit flow π generated by the bank.
- The *equityholders' portion of the switching costs*, denoted by $SC(V,\sigma)$, is the current value of the losses for equityholders that arise from shifting the portfolio risk from σ_L to σ_H or vice versa. In other words, anticipating future portfolio restructuring, the value of the asset portfolio to the equityholders is not V but only $V - SC$.
- The *value of equity*, denoted by $E(V,\sigma)$, is simply the residual value

$$E(V,\sigma) = V - \frac{c}{r} + DI(V,\sigma) + CV(V,\sigma) - SC(V,\sigma). \tag{4}$$

3. Valuing a claim contingent on (V,σ)

The issue in this section is the valuation of a claim contingent on the state of the bank (V,σ). The respective equations will be derived by investigating a general claim $F(V,\sigma)$ which covers all the claims involved in our model as special cases. The adaptation of the general results to the special claims D, CV, and SC is presented in Appendix B. DI and E can then be obtained using Eqs. (3) and (4).

Suppose $F(V)$ is a claim contingent on V and, for a given $\sigma \in \{\sigma_L, \sigma_H\}$, the thresholds V_1 and V_2 ($V_1 < V_2$) are boundaries of a stable regime (see Section 2). That means there are

(i) no switching points,
(ii) no closure points inside these boundaries, and
(iii) the interval (V_1, V_2) either belongs entirely to the 'closure' region ($V_2 \leqslant B(\sigma)$) or is entirely in the 'continuation' region ($B(\sigma) \leqslant V_1$).

Furthermore, this claim provides

(iv) a constant profit flow α as long as the process V is inside these boundaries, and
(v) if the regulator closes the bank at some $\widehat{V}$, the claim pays $\beta + \gamma\widehat{V}$.

Deriving the valuation equations we assume that the two portfolios that span the bank's investment opportunities are traded.[13] Let r denote the constant instantaneous riskless interest rate. Then applying Itô calculus, we find that F has to satisfy the second-order

[13] We make this assumption because we want to analyze how regulation affects risk shifting of banks abstracting from the effects driven by risk preferences of investors. However, we could alternatively assume that only the bank's equity is traded. Then the equity price process reveals the market price of risk which in turn determines the market price of any claim contingent on the banks assets (see, e.g., Björk, 1998, Chapter 10). The model can be solved in a very similar way, e.g. Eq. (5) will change to $rF = \sigma^2 V^2 F_{VV}/2 + (\mu - \kappa\sigma) V F_V + \alpha + \mathbf{1}_{[0,B(\sigma))}\lambda(\beta + \gamma V - F)$, where κ denotes the market price of risk. The results are qualitatively similar but partly driven by the parameterization of the model with respect to the market price of risk and the drift rates of the portfolios.

ordinary differential equation

$$rF = \frac{1}{2}\sigma^2 V^2 F_{VV} + (r-\delta)VF_V + \alpha + \mathbf{1}_{[0,B(\sigma))}\lambda(\beta + \gamma V - F) \tag{5}$$

inside the interval (V_1, V_2), where $\mathbf{1}_{[0,B(\sigma))}$ denotes the indicator function over the interval $[0, B(\sigma))$ and F_V, F_{VV} are the first and second partial derivatives of the claim value with respect to V.

The general solution of this equation, in the case that V is in the closure region, is given by

$$F(V,\sigma) = \frac{\alpha}{r+\lambda} + \lambda\left(\frac{\beta}{r+\lambda} + \frac{\gamma}{\lambda+\delta}V\right) + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)}. \tag{6}$$

Outside this region the solution is

$$F(V,\sigma) = \frac{\alpha}{r} + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)}. \tag{7}$$

The constants $x_1(\sigma)$, $x_2(\sigma)$, $y_1(\sigma)$, $y_2(\sigma)$ are the negative and the positive roots of the characteristic quadratic polynomial of the respective homogeneous differential equation

$$\begin{aligned}
&\frac{1}{2}\sigma^2 x(\sigma)\big[x(\sigma)-1\big] + [r-\delta]x(\sigma) - [r+\lambda], \\
&\frac{1}{2}\sigma^2 y(\sigma)\big[y(\sigma)-1\big] + [r-\delta]y(\sigma) - r.
\end{aligned} \tag{8}$$

Thus, inside an interval of stable regime, the value of the claim F is entirely characterized by (6) and (7), respectively, which are the analytical solutions of the Hamilton–Jacobi–Bellman equation (5). The only unknowns remaining are the two parameters A_1 and A_2 which must be determined by boundary conditions at the boundaries of this interval.

In our model the canonical boundaries which determine intervals of stability are:

- switching thresholds,
- closure thresholds set by the bank's management,
- the boundaries $B(\sigma_L)$ and $B(\sigma_H)$ of the closure region resulting from the regulatory mechanism $\langle\lambda, B(\sigma)\rangle$, and
- the critical value c/r; at this threshold the functional form of the default payoff of the deposit insurance contract changes. Below c/r, the default payoff is $V - c/r < 0$, since the deposit insurance has to bear the difference between the asset value and the face value of deposits. Above c/r, the default payoff to the deposit insurance claim is zero, since the bank's assets value is sufficiently high to cover deposits.

The boundary conditions are derived for the general claim F in Appendix A, and for the specific claims in Appendix B. For given switching and closing thresholds chosen by the bank management, the value of any claim is analytically determined as it is shown in Appendix C.[14] In the next section, the conditions determining the optimal strategy are derived.

[14] We refer to these threshold choices as a choice of operational strategy.

3.1. Optimality conditions

The aim of bank management is to find the operational strategy which maximizes the equity value. As stated in Section 2, the choice variables are the switching points and the exit thresholds which have to be fixed simultaneously. The first-order conditions for switching and closure points that are boundaries of intervals of stability imply smoothness at the respective boundaries.[15]

- If $(\widehat{V}, \sigma_c)$ is a switching point, substitution of the respective boundary condition for D, CV, and SC (see Appendix B) into (4) leads to

$$\lim_{V \to \widehat{V}} E(V, \sigma_c) = E\big((1-k)\widehat{V}, \sigma_{-c}\big), \tag{9}$$

  stating that there is no jump in equity value when the asset portfolio is reorganized. Taking the first derivative of this boundary condition with respect to $\widehat{V}$ leads to the optimality condition

$$\lim_{V \to \widehat{V}} E_V(V, \sigma_c) = (1-k) E_V\big((1-k)\widehat{V}, \sigma_{-c}\big). \tag{10}$$

- If $(\widehat{V}, \sigma_c)$ is the point at which management decides to close the bank, the boundary condition for E is

$$\lim_{V \to \widehat{V}} E(\widehat{V}, \sigma_c) = 0, \tag{11}$$

  leading to the optimality condition

$$\lim_{V \to \widehat{V}} E_V(\widehat{V}, \sigma_c) = 0. \tag{12}$$

Since the optimality conditions (10) and (12) are non-linear, the determination of the optimal thresholds and the verification of the second-order conditions has to be performed numerically.

4. BB versus VaR—comparison of two regulatory approaches

Based on the framework developed in the last two sections, we now consider two stylized regulatory systems, a Basel I building block (BB) approach and a genuine value-at-risk (VaR) based approach. We start with briefly outlining current regulations and then look at the main differences in capital requirements. Finally, we analyze the implications of these regulatory mechanisms on the optimal risk taking behavior of the bank management.

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[15] See Dixit (1991, 1993) for a discussion of the so-called 'smooth pasting conditions.'

4.1. Capital requirements

One of the main ideas of the 1988 Basel Accord (see Basel Committee on Banking Supervision, 1988) is to increase bank soundness by requiring banks to back-up their assets with a prespecified amount of equity capital. In general the capital requirement which should cover credit risk is set to 8% but for asset classes that are considered less risky, like loans to the government and supranational organizations, there exist discounts on the capital requirement.[16] In an amendment to the Basel Accord in 1996 the bank's assets are divided into the trading book, containing all positions intended for short-term resale, and the banking book, that comprises all other assets, especially the loan portfolio.[17] In the same document, capital requirements are also specified for the market risk in the trading book. To fulfill these requirements, banks can either choose a BB method or use their internal VaR models to compute the adequate capitalization. The most recent step in international bank regulation is the New Basel Capital Accord of the Basel Committee on Banking Supervision (2001), which proposes to improve capital adequacy regulation for credit risk. Multiple options are available to the bank to compute the capital requirement for credit risk. These options differ in the granularity of the risk buckets and in the requirements on the banks' internal credit rating systems.

The guidelines of the Basel Committee have been implemented by almost all countries with minor modifications. In our paper, however, we do not want to model a country-specific implementation, but rather theoretically analyze the rationale for the recent trend in bank regulation towards more risk-sensitive capital standards. To formalize this transition, we model two stylized approaches for setting capital requirements. As the starting point, with low risk sensitivity, we consider a simple BB approach, while we use a VaR approach as a framework, where capital is directly linked to asset risk.[18]

The BB approach, which is current practice in almost all countries, is easy to implement. First the assets are assigned to risk buckets and then capital requirements are computed using given weights. Once assigned to a bucket, the asset has the same capital requirement as all others in this bucket. Thus, while banks are penalized by higher capital requirements for inter-bucket risk shifting, such as substituting government bonds with corporate loans, intra-bucket risk shifting is not captured.

We model the BB regulation by focusing our analysis on intra-bucket risk shifting. We assume that the two asset portfolios available to the bank are formed such that the relative proportions of assets in the respective buckets and thus the capital requirement does not change when the bank shifts from one portfolio to the other. This assumption may seem

[16] See e.g. Jorion (2000) or Dewatripont and Tirole (1994) for a comprehensive treatment of bank capital standards. E.g., corporate debt and real estate have a capital requirement of 8%, asset backed mortgage loans require 4%, claims on OECD banks and regulated securities firms require 1.6% and cash and claims on OECD central governments do not have to be backed up. The overall capital requirement of a bank is calculated as a weighted average.

[17] See Basel Committee on Banking Supervision (1996a).

[18] Comparing the pure building block approach to a combination of building block and value-at-risk capital requirements as it is current practice in most countries would give the same principal results but would weaken the effects. Another reason why we look at a pure value-at-risk regulation is that there is an active discussion, whether regulators should accept internal models to compute capital requirements for credit risk.

very stringent at first. However, it is justified by the fact that Basel I provides only four buckets. Since all corporate loans are in the same bucket, regardless of the borrower's rating, the bank can lower the average rating of its corporate loan portfolio without changing capital requirements. While there will always be some heterogeneity in the risk structure of the assets in a bucket, the differences are potentially large under capital regulations that are in the spirit of Basel I. Banks have recognized this weakness in the regulations and are exploiting it. This regulatory capital arbitrage (see Jones, 2000) is seen as an impediment to effective regulatory supervision by many authorities (see Meyer, 1998). In our analysis the principal results also hold when the two portfolios have different capital requirements under the BB approach as long as there is a discrepancy between the regulatory capital requirement and the capital necessary to cover economic risk.

Due to the assumed capital structure (see Section 2), the bank has a simple balance sheet. The assets with current market value V are on the asset side. The liabilities are represented by perpetual deposits with a constant instantaneous coupon of c and face value c/r (where r denotes the instantaneous riskless interest rate) and by equity. The regulator's goal is to preserve a safety cushion, such that the value of the assets V is sufficient to satisfy the depositors' claims c/r. Under the BB regulation, the minimum cushion $V - c/r$ is determined by the risk-weighted assets of the bank. Depending on the bank's borrowers, this capital requirement will be a fraction ρ of the bank's assets. In the case of an audit, the bank will be allowed to continue operations only if the safety cushion is at least as large as the capital requirement:

$$V - \frac{c}{r} \geqslant \rho V. \tag{13}$$

The main feature of the BB approach is that the exposure to a risk factor is limited. For example, for a given amount of capital the notional value of loans a bank can give to the corporate sector is limited. Additional equity capital has to be raised before the bank can grant new loans. The variability of the risk factor is not included in the computation of necessary capital. So, for example, default and recovery rates for loans are not relevant for the capital requirements that apply to loans.[19] According to the assumption that the relative proportions assigned to the building blocks are identical for both portfolios, the fraction ρ does not change when asset substitution takes place. The closure threshold $B(\sigma)$ under the BB regulation is therefore constant:

$$B(\sigma)^{\mathrm{BB}} = \frac{1}{(1-\rho)} \frac{c}{r}, \quad \sigma \in \{\sigma_L, \sigma_H\}. \tag{14}$$

The VaR approach is conceptually different from the BB approach since it includes not only the exposure to risk factors but also the volatility of the risk factors. VaR regulation demands that, in case of an audit, the bank's safety cushion $V - c/r$, the difference between asset value and the face value of debt, must be at least as high as the p% VaR for a time horizon of T multiplied by a 'panic factor' ξ which is set by the regulator.[20]

[19] Another example is equity risk of the trading book. While the maximum amount invested in stocks is limited by the banks capital base, the volatility of the stocks in the banks portfolio is not considered.

[20] Usually p is set to 99% and T is 10 days, the panic factor is set to three in most countries and is intended to cover model risk.

Since the asset value of the bank V follows a geometric Brownian motion (see Eq. (1)), the returns are normally distributed with mean $(\mu - \delta - \sigma^2/2)T$ and a standard deviation of $\sigma\sqrt{T}$. The factor T scales the moments of the distribution. So, for example, if μ, δ, and σ are measured with respect to the time unit of one year ($=250$ trading days), we have $T = 10/250$ to capture the risk over the next ten days. After linearizing and neglecting the mean of the distribution, as is done in most VaR implementations, the p% quantile of the loss distribution is given by $\Phi^{-1}(p)\sigma V\sqrt{T}$, where $\Phi^{-1}(p)$ is the p% quantile of the standard normal distribution. Or in other words, the bank is allowed to continue its operation if

$$V - \frac{c}{r} \geqslant \xi a \sigma V, \quad \text{where } a = \Phi^{-1}(p)\sqrt{T}. \tag{15}$$

The closure threshold for VaR regulation is, thus, given by

$$B(\sigma)^{\mathrm{VaR}} = \frac{1}{(1 - \xi a \sigma)} \frac{c}{r}. \tag{16}$$

Comparing Eqs. (14) to (16), we can see that the main difference between the two regulatory regimes is that VaR regulation explicitly accounts for the risk of the portfolio by adjusting the capital requirements, whereas the BB regulation is independent of the volatility of the institution's assets if risk shifting occurs within buckets.

Despite the broad consensus that capital requirements should be more risk sensitive, several concerns about VaR-based capital requirements have been raised in the literature. First, Basak and Shapiro (2001) find that under a VaR constraint, asset managers only partly insure their portfolios against losses. In particular, the bad states of nature remain entirely uninsured. In their model the VaR constraint has to be satisfied at some final time T, allowing managers to continuously readjust their portfolio. And it is particularly this adaptation of the portfolio before time T that reduces wealth in the bad states. However, regulators have recognized this problem, and in order to get a picture of the instantaneous portfolio risk, the VaR horizon for back testing for actively traded assets is explicitly set to one day (see Basel Committee on Banking Supervision, 1996b). To capture this idea and to adequately model the institutional features of bank supervision, we focus on the pure instantaneous VaR which is proportional to the portfolios volatility. A second criticism pointed out (see Kupiec, 1995) concerns the accuracy of risk measurement in light of the unobservability of the volatility of the bank's assets. The main problem for the regulator is not rejecting wrong VaR reports (type II error). Recognizing that this can allow an undercapitalized bank to go undetected, the audit intensity λ has to be adjusted to incorporate this risk in our model. The third concern about the accuracy of VaR measurements stems from possible non-normality of the portfolio returns (for example, fat tails). While we have to make the assumption of normally distributed returns to keep the model analytically tractable, the capital requirements can be adjusted to a proportional error in VaR measurement by adapting the panic factor ξ.[21]

[21] As outlined in Basel Committee on Banking Supervision (1996a), this multiplier is "designed to account for potential weaknesses in the modeling process" such as fat tails in the distribution of risk factor returns, sudden changes in volatilities and correlations, intra day trading, event risk and model risk (especially with options).

4.2. Risk-shifting incentives

The risk-taking incentive that leads bank managers to increase an institution's risk stems from the fact that the deposit insurance corporation gives the equityholders a put option on the bank's assets. The value of this put option increases with the volatility of the underlying asset and thus makes higher risk desirable to equityholders.[22] To mitigate this problem, different regulatory responses have been proposed, all of them focusing on resolving the convexity in the value function of equity. Fries et al. (1997) suggest state-dependent subsidies and equity support schemes to make the equity function linear for troubled banks. Bhattacharya et al. (2002) choose the closure threshold and the auditing intensity such that the value function is linear for solvent banks (i.e., for banks whose asset values satisfy the minimum capital requirement). Rochet (1992) shows that limited liability creates an incentive which leads even the risk-averse bank (a bank that behaves like a portfolio manager that tries to maximize expected utility) to pursue a very risky investment strategy. He suggests minimum capital requirements (a closing rule) to overcome this undesirable behavior.

Due to regulatory intervention, equityholders are not entirely free in setting the optimal closure point for the bank with the consequence that they cannot fully exploit the benefit of the put option. Depending on λ and $B(\sigma)$ the risk-taking incentive is weakened or managers might even find it beneficial to reduce asset risk.

Under the BB regulation, the auditor's toughness (i.e., choosing a high λ) is the key instrument for mitigating risk taking. Figure 1 illustrates the impact of different audit intensities on the equity value by means of an example. When λ is low, the convex shape of the simple put option prevails over the entire range of the underlying asset payoff, which means that equityholders have a global incentive to take risk. However, under strict auditing, the curvature of the equity value changes its sign. When the asset value is significantly below the closure threshold, an audit will result in the immediate closure of the bank. Since higher asset volatility increases the chance that the bank recovers before the next audit takes place, it is preferred to low volatility. In other words, if the bank is in real distress, bank management has a strong incentive to gamble for resurrection, regardless of the audit intensity. When the capital requirement is met, there is still the positive effect of high volatility on the equity value that stems from exploiting the deposit insurance system. However, high volatility increases the probability that the bank runs into distress (i.e. the asset value drops below the closure threshold) and that it will, due to auditing, be closed by the regulator. This harms the equityholders, who lose the charter value of the bank. If λ is sufficiently high, the negative effect of high volatility on equity dominates the positive effect, and the well-capitalized bank prefers low risk to high risk. When well-capitalized banks reduce their assets' risk, they essentially lower the deposit insurance corporation's liability (see Section 5 for a more detailed discussion of this feature). Nevertheless, whether the bank managers really switch the risk level, and when they optimally do it, also depends on the

[22] This fact is well documented in previous research such as Matutes and Vives (2000) who show in a model of bank competition that flat rate deposit insurance will induce banks to take maximum asset risk. Implementing risk-sensitive deposit insurance pricing in an asymmetric-information setting can be problematic, however. See Chan et al. (1992).

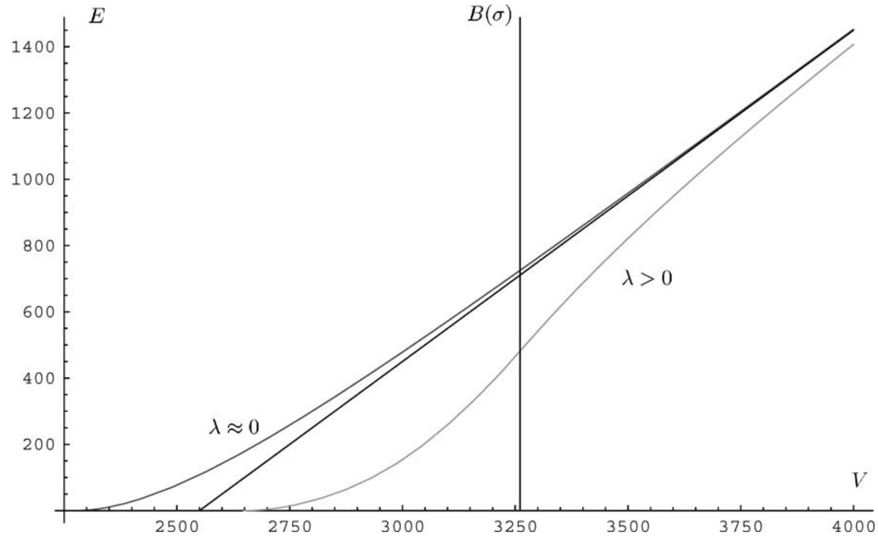

Fig. 1. Bank equity value E under Basel I building block regulation as a function of the asset value V for high and low audit intensities plotted against the asymptote. The vertical line represents the minimum capital requirement. While convexity prevails for low audit intensities, high audit intensities create an incentive to reduce risk for the solvent bank. The face value of debt is assumed to be 3000.

costs for rearranging the portfolio and can only be answered after analyzing the model with a particular parameterization, i.e., under full consideration of higher order effects, as done in Section 5.

Under VaR regulation, the trade-off between exploiting deposit insurance and fearing closure due to regulatory enforcement is still valid. However, VaR regulation enhances the incentive for solvent banks to reduce risk by setting the minimum capital requirement according to the actual asset risk. Since higher asset volatility implies higher capital requirements (see Eq. (16)), a bank can improve its capital ratio by reducing asset risk. The effect of risk-sensitive capital requirements is most evident in the case where the asset value is between the closure threshold for low risk $B(\sigma_L)$ and the closure threshold for high risk $B(\sigma_H)$. If an audit occurs and the bank is invested in the low-risk portfolio, the audit confirms solvency, i.e., no negative consequences for the bank. In the same situation, if the bank's portfolio consists of high-risk assets, an audit results in bank closure. Due to the diffusion-nature of the asset value process, this effect creates an incentive for rational equityholders to reduce asset risk even for the well-capitalized bank (i.e., $V > B(\sigma)$). In other words, by switching to the low-risk portfolio, the bank can enhance its capital ratio and simultaneously reduce the probability of getting into financial distress.

Extending the existing continuous time models on risk taking in the context of banking regulation, we explicitly allow bank managers to respond to the identified incentives by actually restructuring the bank's portfolio or shutting down the bank:[23]

[23] We allow the bank to respond to the identified incentives in the form of control limits policies, which are optimal in the case of lump-sum costs associated with controlling Brownian motion, see Section 2. However, we

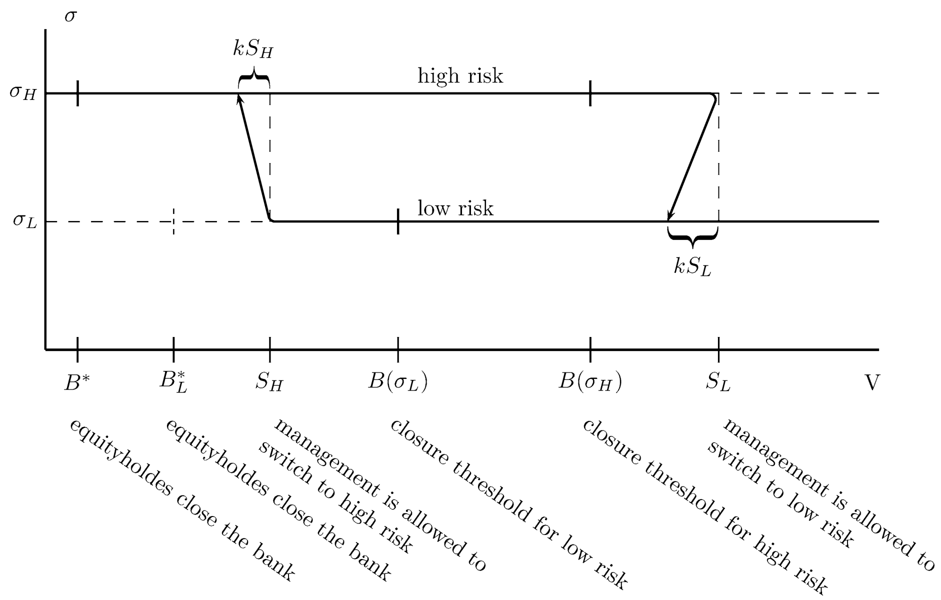

Fig. 2. Management's risk taking behavior under value-at-risk regulation when the bank implements the *full-hysteresis* strategy, i.e., switching to high risk when in distress and to low risk when sufficiently capitalized. Asset substitution destroys a fraction k of the asset value at every switch.

- The low-risk bank is allowed to respond to declining asset value V by switching to the high-risk portfolio at a threshold S_H. It substitutes the high-risk portfolio for the low-risk portfolio, incurring the proportional switching costs kS_H. Alternatively, the management of the low-risk bank may directly make use of limited liability and find a lower threshold B_L^* where it closes the bank voluntarily.
- Similarly, the well-capitalized high-risk bank is allowed to respond to growing asset value by switching to the low-risk portfolio at an upper boundary S_L, again incurring the proportional costs kS_L. Furthermore, the high-risk bank's management can again close voluntarily at some lower threshold B^*.

Figure 2 illustrates the available choices in risk shifting and introduces the critical levels where the bank can switch the asset risk or where it closes voluntarily. The switching costs, which form a deadweight loss, are responsible for the fact that the bank's possible states form a hysteresis. For banks implementing a switching strategy, the correspondence between asset value and asset risk is non-unique. According to the available alternatives, the bank's management has the choice between four qualitatively different strategies.

restrict the set of available strategies in the sense that we do not explicitly regard choices that do not conform with the identified incentives, e.g., we do not allow that a well-capitalized bank has the opportunity to switch back and forth between the portfolios at some arbitrary thresholds.

- *No-substitution:* the bank does not change asset risk but only utilizes limited liability to extract wealth from the deposit insurance.
- *Risk-reduction:* the high-risk bank switches to the low-risk portfolio when it is sufficiently capitalized, the low-risk bank sticks to its portfolio; when running into distress, the bank makes use of limited liability.
- *Gambling-for-resurrection:* the high-risk bank sticks to its portfolio and defaults at some lower boundary, the low-risk bank switches to the high-risk portfolio when it runs into distress.
- *Full-hysteresis:* the bank responds to both risk shifting incentives by switching to high risk in distress and to low risk when it is well-capitalized.

Optimal decision. When implementing one of these four strategies, the bank chooses the location of the switching and closure thresholds optimally to simultaneously satisfy the optimality conditions of Section 3.1. The decision about which of these four fundamental strategies the bank should choose is done on the basis of equity-value maximization. We will explore the optimal choice of the bank with respect to different regulatory environments in Section 5.1 after deriving some comparative statistics results.

Figure 3 shows the bank's equity value as a function of the asset value when the bank optimally implements the *full-hysteresis* strategy. Despite the convexity of the high-risk value function, the VaR-based capital requirements (together with an appropriate λ) create enough incentive for the well-capitalized bank to switch back to low risk.

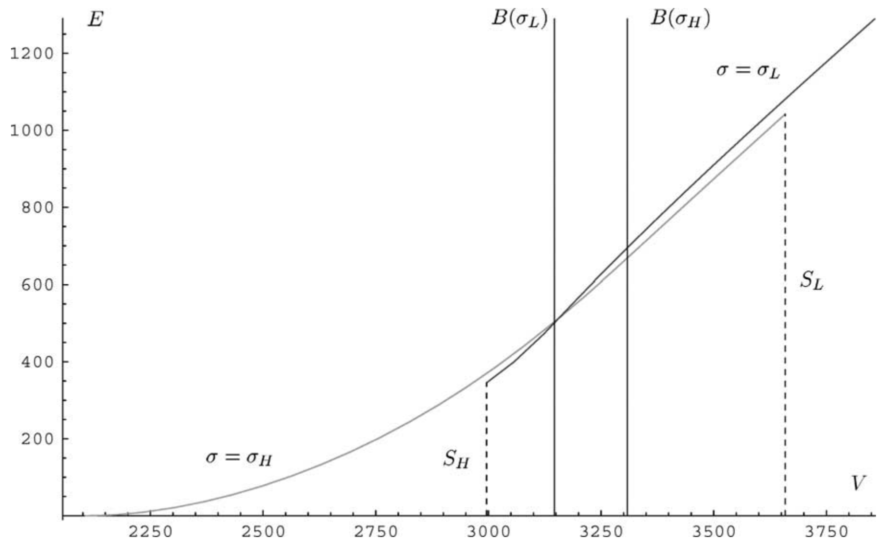

Fig. 3. Bank equity value E under value-at-risk regulation as a function of the asset value V when the bank implements the *full hysteresis* strategy. The vertical lines represent the closure thresholds for the low-risk and the high-risk portfolio. The two functions show the equity value for high risk ($\sigma = \sigma_H$) and low risk ($\sigma = \sigma_L$), respectively. While the bank prefers high risk when it is insolvent, it reduces risk when sufficient solvency is regained.

5. Results and comparative statistics

In this section we analyze the different incentives and potential benefits created by the BB and VaR regulations by means of a numerical example. For this purpose we first take a closer look at the mechanics behind the optimal risk choice and derive some comparative static results. Secondly, we analyze these consequences of different risk-taking behavior on the deposit insurance agency and the bank's equityholders. Finally, we derive some policy implications. Unless otherwise stated, we take the parameter values from Table 1. Note that we chose the panic factor to be one for the base case. This is because the panic factor is intended to capture model risk, which is not existent in our model. Nevertheless, we examine the general effect of a panic factor greater than one on the risk-shifting behavior of banks.

As argued in the previous section, the risk-reduction incentive under BB-based capital requirements is weaker than under VaR regulation. To demonstrate this feature, the parameter set of the base case (Table 1) is chosen such that it is optimal for the solvent bank to reduce risk when it is VaR-regulated, and to stick to high risk when it is BB-regulated.[24]

Table 1
Parameter values for the numerical analysis and results of the base case scenario

Panel A. Parameter values		
Coupon of debt	c	150
Riskless interest rate	r	0.05
Face value of debt	c/r	3000
Excess cash flow generated by bank	$\pi_H = \pi_L$	22.5
Audit frequency	λ	0.45
Return volatility of low-risk portfolio	σ_L	0.1
Return volatility of high-risk portfolio	σ_H	0.2
Switching costs	k	0.01
Cash flow rate	$\delta_L = \delta_H$	0.01
Capital requirements—BB regulation	ρ	8%
Value-at-risk confidence level	p	99%
Value-at-risk holding period	T	10 days
Panic factor	ξ	1
Panel B. Regime switching points for the VaR-regulated bank		
Equityholders abandon bank	B^*	2105.31
Closure threshold—low risk	B_L	3146.63
Closure threshold—high risk	B_H	3308.34
Managers switch to high risk	S_H	2995.94
Managers switch to low risk	S_L	3658.62
Panel C. Regime switching points for the bank with BB capital requirements		
Equityholders abandon bank	B^*	2100.66
Closure threshold	B_L	3260.87
Managers switch to high risk	S_H	3191.50

[24] To be accurate, if the bank is established as a well-capitalized low-risk bank, then it will not switch to the high-risk portfolio immediately. However, once the BB-regulated bank has reorganized its portfolio at S_H it will stay a high-risk bank.

In the following comparative statistics we will explore under what circumstances other strategies will be selected by the bank.

5.1. Comparative statistics

In Fig. 4 the locations of the critical thresholds for the VaR regulated bank are plotted for different levels of the volatility of the risky portfolio (σ_H). We see that the closure threshold set by the regulator for the high-risk portfolio, $B(\sigma_H)$ increases with the portfolio's risk. Looking at the equityholders' optimal closure decision, we see that a higher volatility gives banks a greater value for the gambling for resurrection strategy by increasing the probability that the asset value will grow beyond the closure threshold again within the foreseeable future. As this effect is not compensated by the increase in the closure threshold $B(\sigma_H)$, which is approximately linear for small changes in the financial institution's risk, bank equityholders are willing to support the bank for a longer period of time (B^* decreases). The increased attractiveness of gambling also makes it more advantageous for low-risk banks to start gambling by switching to high risk at point S_H once the bank is undercapitalized (i.e., the asset value is lower than $B(\sigma_L)$). The switching point S_L, where high-risk banks switch to low risk again, is substantially increasing with σ_H. The value of the deposit insurance put option increases with volatility. This effect dominates the gain from reduced insolvency risk when switching to low risk and the reward in form of lower capital requirements. It is interesting to see that beyond a certain level (which is ≈ 0.23 in our example), equityholders see no reason to switch back to low risk any more. Lower capital requirements cannot offset the high value of the deposit insurance option. Under the

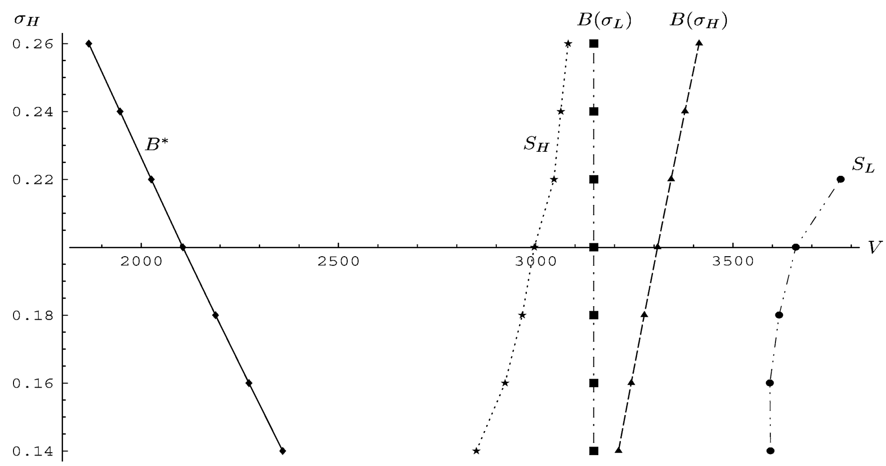

Fig. 4. Locations of the critical thresholds for different values of the high-risk technology's volatility σ_H. Higher risk increases the chances of an insolvent bank to regain solvency and makes equityholders keep the bank alive for a longer period of time (B^* decreases). Higher risk also makes banks switch to high risk earlier at S_H. And it makes managers switch back to low risk (S_L) later, as the option value of the deposit insurance decreases with volatility. If $\sigma_H \gtrapprox 0.23$, equityholders will not switch back to low risk any more.

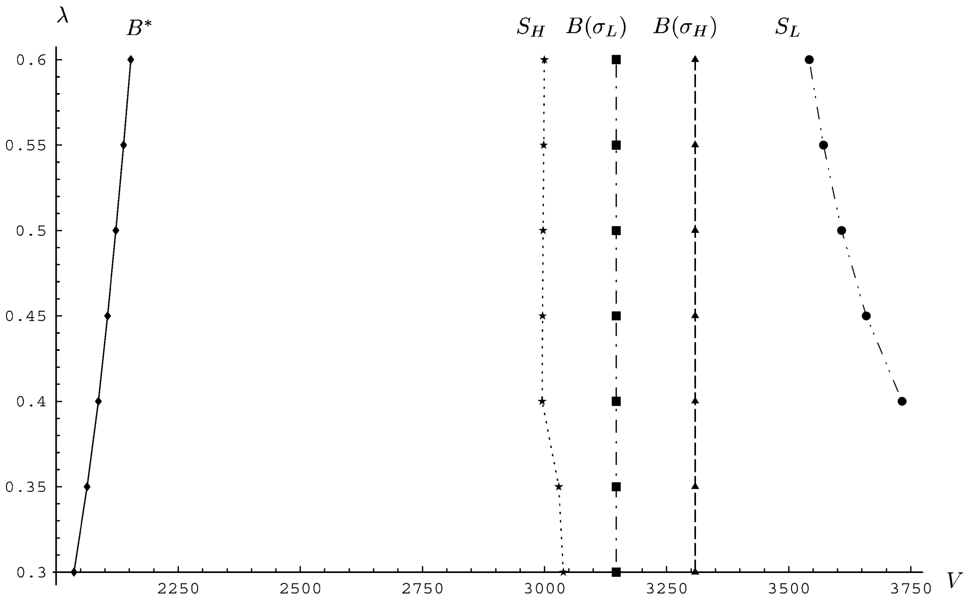

Fig. 5. Locations of the critical thresholds for the VaR regulated bank for different values of audit frequency λ. While the closure thresholds ($B(.)$) are not affected by the auditing intensity, a tougher regulator makes it less attractive for shareholders to support an ailing bank (B^* increases). For high audit intensities, the high-risk portfolio also becomes less attractive, managers switch earlier back to low risk (S_L decreases). If auditing is too relaxed (here below $\lambda \approx 0.37$), bank managers do not have an incentive to switch back to low risk any more.

current parameter set, the BB-regulated bank has no incentive to reduce risk, independent of the investment opportunity σ_H. That is, it finds it optimal to stick to high risk even if it is well-capitalized.

We see that the investment opportunities of the bank have a substantial impact on risk-taking incentives. If banks can increase their risk substantially, the incentive for solvent banks to reduce risk is destroyed. One way for the regulator to maintain the risk-reduction incentive for higher volatility levels as well is to increase the auditing intensity, which will be explored next.

Figure 5 illustrates the impact of the auditing frequency λ. The closure thresholds $B(\sigma_L)$ and $B(\sigma_H)$ are not affected by the auditing policy of the regulator. The regime switching points display behavior that is consistent with intuition. Equityholders are less willing to support an insolvent bank as the probability of an audit increases. The critical asset value B^* at which the equityholders will close the bank therefore increases with the audit frequency. The switching point S_H, where equityholders switch to high risk and start to gamble is determined by two offsetting effects. If the value of the banks assets is below the closure threshold $B(\sigma_L)$, a higher probability of an audit makes it more likely to get caught in the closure region. A more stringent auditing policy therefore puts additional pressure on management to start gambling. But once the switching to higher risk has occurred, capital requirements increase, and these capital requirements are harder to meet before the next audit. This effect, together with deadweight switching costs,[25] determines the location of the switching threshold S_H. In this example the two effects approximately offset each other.[26] A similar trade-off determines the location of the point S_L, where the manager switches from high risk back to low risk. On the one hand, the switch reduces the value of the deposit insurance option, resulting in a value gain from switching late (i.e., switching at high V). On the other hand, switching allows the manger to be more relaxed, since the capital requirements are lower, i.e., the distance to the closure region increases and the probability of getting into trouble decreases. As the regulator becomes tougher (λ increases), the latter effect dominates and managers have an incentive to switch at lower values of V. If the auditor reduces λ below a certain level λ_{min} (which is ≈ 0.37 in our example), an audit is so unlikely that banks will focus on exploiting the deposit insurance option instead of switching back to low risk. Under the BB regulation, the minimum audit intensity that creates a sufficient risk-reduction incentive is significantly higher ($\lambda_{\text{min}} \approx 0.51$). Thus, VaR-based capital requirements create a stronger incentive to reduce asset risk, or in other words, it requires less effort of the supervising authority to enforce prudent behavior. This finding supports the results of Rochet (1992) who points out that capital requirements with market-based risk weights implement efficient risk choices by banks.

From the comparative statistics we can see how different regulatory parameters affect the optimal strategy of the bank. With regard to the different strategies outlined in Section 4.2, we find that the *no-substitution* strategy is optimal only when the costs of asset substitution k are high or when the difference in risk $\sigma_H - \sigma_L$ is low. Then management will refrain from asset substitution because it destroys a large fraction of asset value compared to the gain from changing asset volatility. The bank will stick to the given volatility and the only strategic element is the threshold where the bank is closed voluntarily. The *risk-reduction* strategy is optimal when the audit intensity λ is very high and the capital requirements for both portfolios are similar (e.g. BB requirements). Then it pays to reduce asset risk when the bank is well-capitalized because this reduces the probability of distress. However, if the bank is in distress and λ is very high, the bank will be audited and closed with high probability independent of the volatility. The *gambling-for-resurrection* strategy is optimal when λ is moderate and/or the risk sensitiveness of the capital requirement is low. In this case, it is optimal to take risk when in distress, because it increases the probability to re-gain solvency. The moderate auditing intensity, combined with the low risk sensitivity of the capital requirement, prevents banks from reducing risk when they are well capitalized. The *full-hysteresis* strategy is optimal when λ is sufficiently high and/or the risk sensitiveness of the capital requirement is sufficiently large. In this case both the risk-taking and the risk-reduction incentives are large in order to make this strategy optimal, even though switching costs form a deadweight loss.

[25] Since switching costs are assumed to be proportional to the asset value there is a general incentive to switch at low asset values. However, to reduce the switching frequency, decision makers try to increase the distance between the switching points S_L and S_H.

[26] While in this section the intuition is explained using only first order effects, actually all future switching decisions and all future switching costs are incorporated in the equityholders' optimization.

5.2. Deposit insurance and equity value

Once the regulator has specified the regulatory mechanism, the bank's equityholders will respond by choosing an optimal risk-taking policy. The bank's strategy is crucial for evaluating the liability of the deposit insurance corporation. Since deposit insurance guarantees the face value c/r to the bank's depositors, the current value of the potential future liability (DI) of the deposit insurance corporation is given by the difference between the face value and the current market value of deposits. Figure 6 shows the regulator's liability as a function of V for two banks following different strategies. The VaR-regulated bank switches to the low-risk asset portfolio when it is well capitalized, while under the BB regulation the bank finds it optimal to always stick to high risk. The bank which adopts the switching strategy reduces the risk in the banking sector, and thus lowers the liability of the deposit insurance fund for all asset values. The chosen regulatory mechanism has an impact on the deposit insurance system, as it influences the risk-shifting strategies adopted by financial institutions. Independent of V, the regulator's liability is lower when implementing a regulatory mechanism that encourages risk reduction. The regulator can either keep the surplus or it can significantly lower the insurance premia which will directly benefit the bank's equityholders.[27] Troubled banks, however, are still a problem under both regu-

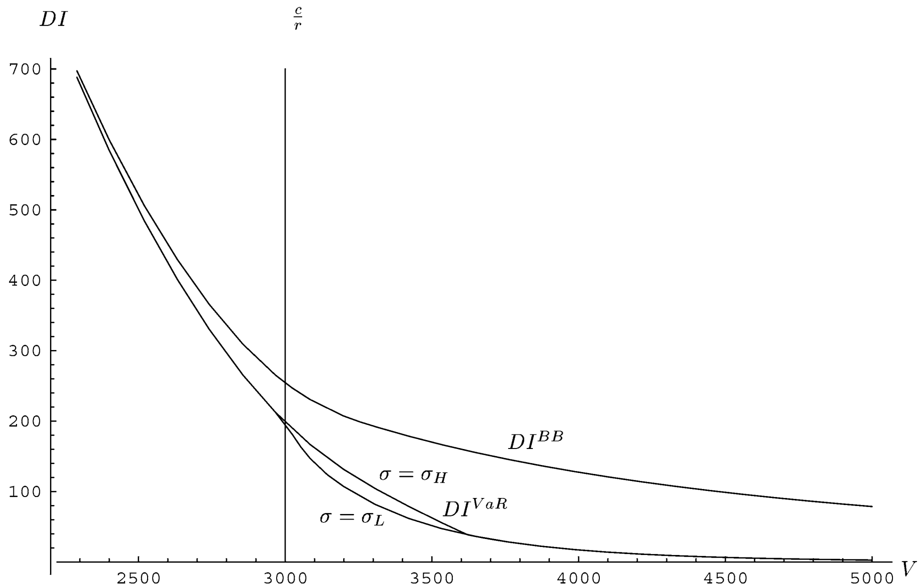

Fig. 6. The current value of the potential liability of the deposit insurance corporation for different levels of V under BB and VaR regulation. For both choices of asset risk the regulator's liability under VaR regulation is below the one under BB regulation. Under VaR regulation, the deposit insurance claim forms a hysteresis, i.e., depending on history bank managers either choose high risk σ_H or low risk σ_L.

27 We have not included an insurance premium in our model, but if the regulator charges the bank an up-front premium, the fair value is DI, which is lower under VaR regulation. The same intuition applies for a continuously

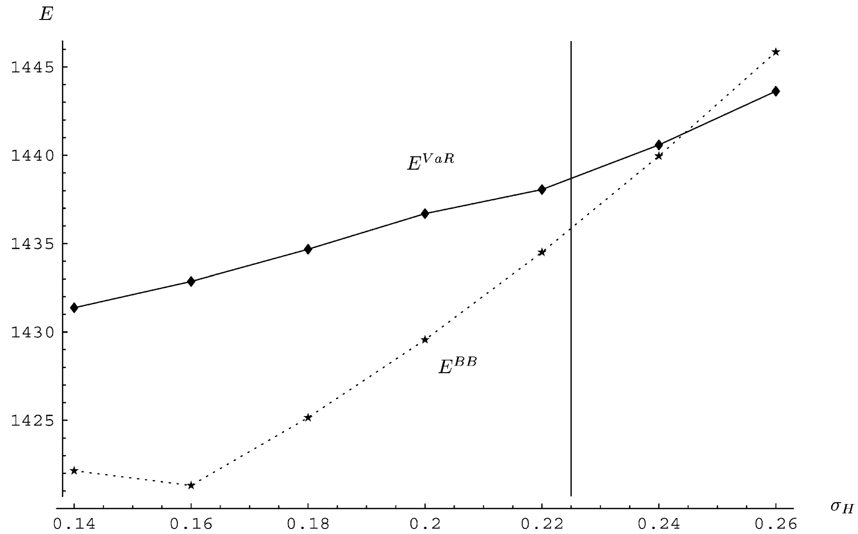

Fig. 7. Equity value of the solvent bank ($V = 4000$) as a function σ_H of the volatility of the high-risk portfolio. VaR regulation gives solvent banks a higher equity value than regulation according to the BB approach as long as they have an incentive to reduce risk (left of the vertical line). When sticking to high risk is optimal, banks may be better off with BB regulation, because of lower capital requirements.

latory regimes as the regulator's liability increases sharply when banks come into financial distress.

In regard to the impact of the regulatory regime on the equityholders' claim, one expects that the reduction in the deposit insurance liability due to risk reduction is at the expense of the equityholders. Looking again at Eq. (4), it seems obvious that a decline in the value of the deposit insurance reduces the equity value:

$$E(V,\sigma) = V - \frac{c}{r} + DI(V,\sigma) + CV(V,\sigma) - SC(V,\sigma).$$

However, a change in capital requirements also has consequences for the value of the bank charter CV and the switching costs SC. Therefore, equityholders will vote in favor of VaR if and only if the increase in the charter value (due to an increase in expected life time of the bank) outweighs the loss in equity value that stems from reduced deposit insurance value and (possibly) increased switching costs. Solvent banks and regulators may thus have common interests. Banks want to reduce risk to increase the value of their charter while the regulator desires a sound banking system. VaR regulation makes it easier to bring these incentives in line, by rewarding low risk banks with lower capital requirements.

Figure 7 shows the equity value for the solvent bank ($V = 4000$, base case scenario and different values of σ_H) under both, VaR regulation (optimal strategy is *full-hysteresis*

paid premium. A more sophisticated premium schedule that considers the risk level of the bank's portfolio after each audit would strengthen the risk-reduction incentive. The premium-determination process in our setting is made easier by the fact that we do not consider the asymmetric-information problems that can frustrate the determination of fairly priced deposit insurance (see Chan et al., 1992).

as long as $\sigma_H \lessapprox 0.23$) and BB regulation (optimal strategy is *gambling-for-resurrection*) for equal audit intensity ($\lambda = 0.45$). Exploiting the deposit insurance system is not that attractive for well-capitalized banks, since the put option is far out of the money. In this situation the increase in the charter value outweighs the change in deposit insurance and in the switching costs.

However, if the 'average capital requirement' under VaR regulation is too high compared to BB regulation (e.g., due to a high panic factor ξ) or if gambling is too attractive (due to a high value of σ_H), the change in the value of deposit insurance when moving from BB to VaR regulation dominates and equity holders will vote against VaR regulation. Figure 7 supports this fact. When σ_H is high, it is more attractive to exploit the deposit insurance put option ceteris paribus. In this case well-capitalized banks will not reduce their asset volatility and will thus prefer BB regulation. Many regulators (e.g., in the EU countries) allow banks to choose the regulatory framework for the trading book. According to our model, we should find that financially-sound banks will vote in favor of VaR while troubled banks will stick to BB regulation.

5.3. *Prudent regulation*

We have focused on the optimal response of the bank to a given regulatory mechanism, where we assume that the bank's decisionmakers maximize equity value. Knowing the optimal reaction of a bank to a given regulation, it would be very interesting to derive an optimal regulatory framework, modeling the entire game between policymakers and banks. This, however, requires a specification of the regulator's value function. To do so, one must include the social value of the banking system, including the value of the payment system, welfare-increasing projects that would not be funded by capital markets, and so on, and balance this value against the social costs of bank supervision arising from deposit insurance and auditing and the social costs of bankruptcy arising from direct bankruptcy costs, systemic risk considerations, loss of confidence in the banking system, and so on. To quantify these effects is beyond the scope of this paper and is therefore omitted. Nevertheless, our analysis allows us to explore the influence of the regulatory mechanism on certain components of social welfare.

If one abstracts from the social costs of bankruptcy, deposit insurance is just a welfare-neutral transfer of a liability from the bank to the deposit insurance corporation. The only welfare effects stem from the extra value generated by the bank (reflected in the charter value) and from switching costs (which are a deadweight loss). In this case, maximizing social welfare corresponds to maximizing $CV - SC$. Therefore, the bank's shareholders will vote in favor of the socially-optimal regulatory framework if the deposit insurance fee is fair (see Eq. (4) and the discussion in Section 5.2). Whether BB or VaR is socially optimal depends on the actual model parameters. However, risk reduction of the well-capitalized bank usually increases charter value and the risk-reduction incentive is greater under the VaR regulation, so that in many cases VaR dominates BB regulation (again, see the discussion in Section 5.2).

When one considers the social costs of bankruptcy, welfare is not entirely characterized by $CV - SC$, but, for several reasons, negatively related to the magnitude of the deposit insurance liabilities DI. One reason for this is the possible presence of deadweight losses

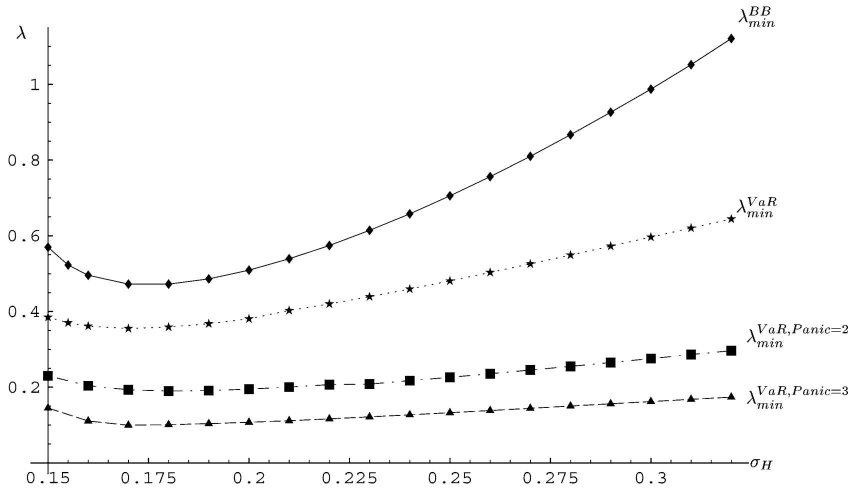

Fig. 8. Minimum audit intensity $\lambda_{\min}$ required to maintain the switching incentive under different regulatory systems for different volatilities σ_H of the high-risk technology. Under BB regulation, the necessary auditing level increases sharply with the volatility of the high-risk technology. Under VaR regulation, the regulator's awareness is less sensitive to the investment opportunity set of the bank, especially if a panic factor is included.

in the deposit insurance system. For example, when a certain fraction of the premium paid has to be used to cover administrative expenses, a smaller insurance system is more efficient.[28] Another reason is that the social costs of bankruptcy may be proportional to the shortfall upon liquidation, i.e., the amount by which the liabilities exceed the value of the assets. Since the deposit insurance claim DI denotes the present value of the future shortfall, social costs are proportional to DI and can be reduced when providing a sufficient risk-reduction incentive.

Hence, taking as a given that it is socially beneficial that solvent banks reduce their asset risk, we ask whether a specific regulatory mechanism induces this behavior. From Section 5 we know that a certain minimum audit intensity $\lambda_{\min}$ is required to provide this incentive for both BB and VaR regulation. Figure 8 compares the minimum level of auditing that has to be performed in order to give solvent banks an incentive to reduce risk for different investment opportunity sets σ_H. According to the discussion in Section 4, the weaker risk-reduction incentive provided by BB regulation transforms into a higher minimum audit frequency $\lambda_{\min}$ compared to VaR regulation. Since auditing costs form deadweight losses, a reduction in the required audit intensity reduces undesirable externalities and increases social welfare. From a social planner's perspective, we might again favor capital requirements based on VaR rather than on BB since the former requires less auditing.

We can also see from Figure 8 that $\lambda_{\min}$ depends on the bank's investment opportunity set. For high values of σ_H, exploiting the deposit insurance option is very tempting for the bank, resulting in a positive slope of $\lambda_{\min}$. That means, to maintain the risk-reduction

[28] The FDIC's budget for administrative expenses in 2000 was 1.18 billion dollars.

incentive the regulator has to apply a higher audit intensity when banks can invest in very risky portfolios. Because of the lack of sensitivity to economic risk, this increase in $\lambda_{\min}$ is more pronounced under BB than under VaR regulation.[29] Thus, when the supervisory authorities are not informed about the bank's investment opportunities ex ante, they may either apply too much auditing, thereby wasting resources or apply insufficient auditing in order to maintain the switching incentive. For example, applying the FDIC's current auditing policy, involving inspections every 12–18 months to the base case scenario under BB regulation, gives well-capitalized banks an incentive to reduce risk only if the volatility of their high-risk portfolio is less than 25%–30%. Our finding about the necessity of a prudent auditing policy supports the decision of the Basel Committee to recognize auditing as one of the main pillars of the new accord. In terms of robustness and to facilitate the calibration of the regulatory mechanism, it makes sense to specify capital requirements that allow the regulator to apply a uniform audit intensity for all banks, independent of specific investment possibilities. This can be achieved by introducing a panic factor $\xi > 1$ for VaR regulation. The capital requirements are affected by this in two ways. First, they jointly increase, and second, they become more risk sensitive (because $\partial^2 B(\sigma)/\partial\sigma\partial\xi > 0$). In Figure 8 we see two effects when moving from BB to VaR regulation: a general drop in the minimum audit intensity and a reduced sensitivity to changes in the investment opportunity set (especially for $\xi > 1$). As pointed out earlier, the official rationale for the panic factor is to cover model risk. However, our analysis demonstrates that a further benefit of the panic factor greater than one is an increased risk sensitivity of the regulatory framework, which then makes VaR regulation more robust in the sense that the auditing behavior of the supervisor does not have to be very precisely fine-tuned to the bank's risk-shifting possibilities.

6. Conclusion

The proposal on the New Capital Accord of the Basel Committee on Banking Supervision (2001) is the most recent important step in an ongoing regime change in international bank regulation. Simple rules of capital adequacy are replaced in order to make required capital more sensitive to the financial institution's risk, thereby closing the gap between regulatory and economic capital. We have provided a theoretical justification for this trend in bank supervision and rigorously analyzed the impact of risk-sensitive capital requirements on banks' optimal risk-taking behavior. We choose a modeling approach, where banks are allowed to switch between two asset portfolios with different volatility. This explicit treatment of the risk-shifting process permits a comparison of regulatory mechanisms that are based on asset value, like the Basel I building block approach, and risk-contingent regulations, like value-at-risk-based capital requirements.

[29] Interestingly, we also see higher minimum audit intensities when σ_H is low. This simply stems from the fact that the gain from reducing risk decreases as the difference of the two portfolio volatilities gets smaller, whereas the switching costs are assumed to be constant with respect to volatility. Again, this effect is more evident under BB than under VaR regulation.

We find that neither the BB nor the VaR regulatory mechanism generally prevents banks from switching to high risk when they are in distress. However, under VaR regulation, well-capitalized banks have a stronger incentive to reduce asset risk than under BB regulation. This is driven by the reward in form of lower capital requirements for low-risk banks.

This reduction of risk decreases the current value of the deposit insurance liability while it increases the current value of the bank charter. Thus, shifting from the Basel Accord BB approach to the risk-based VaR regulation may benefit both the regulatory authority and the equityholders of banks.

While VaR-based regulation gives stronger risk-reduction incentives to banks, it also requires less auditing effort to maintain the risk-reduction behavior. Furthermore, under VaR regulation, this risk-reduction behavior is less sensitive to a change in the bank's investment opportunity set.

Our findings provide support for the current regulatory move toward more risk-sensitive capital requirements. Our analysis also specifies capital requirements and auditing as important pillars of the new regulation, and highlights the importance of considering their interaction, as recognized in the Basel II proposal.

Acknowledgments

We thank Günter Strobl for valuable discussions in the start up phase of our work and Engelbert Dockner, Helmut Elsinger, Bernd Engelmann, Thomas Gehrig, Rajna Gibson, Reint Gropp, Robert Heinkel, Philippe Jorion, Grzegorz Pawlina, Jean Charles Rochet, Benoît Roger, Duane Seppi, Alex Stomper, Neal Stoughton, Ernst-Ludwig von Thadden, Anjan Thakor, Josef Zechner, an anonymous referee and the participants of the American Finance Association Meetings 2001, the European Finance Association Meetings 2001, the Asian Pacific Finance Association Meetings 2001, the Northern Finance Association Meetings 2000, the German Finance Association Meetings 2000, the Bank of Finland/CEPR Workshop 2001 on Moral Hazard Issues in Banking, the BIS/CEPR/JFI Workshop 'Basel II: An Economic Assessment' 2002 as well as of the CCEFM Seminar 2000 in Rust for valuable suggestions and comments.

Appendix A. Boundary conditions

This section derives the boundary conditions for the general claim F. As an abbreviation for Eqs. (6) and (7), we write

$$F = F(V, \sigma; A_1, A_2). \tag{A.1}$$

(1) *Switching threshold*. Suppose $\sigma_c \in \{\sigma_L, \sigma_H\}$ denotes the volatility at the current risk level and σ_{-c} is the volatility at the alternative risk level. Furthermore, let V_i be a switching threshold set by the bank's management at which the assets are reorganized into a portfolio with volatility σ_{-c}. Let $F(V, \sigma_c; A_1, A_2)$ denote the market value of the claim prior to the volatility shift at V_i and $F(V, \sigma_{-c}; A_1', A_2')$ the claim value subsequent to the volatility shift in a neighborhood of $(1-k)V_i$ (according to the convention (A.1)).

Market equilibrium requires

$$\lim_{V \to V_i} F(V, \sigma_c; A_1, A_2) = \begin{cases} F\big((1-k)V_i, \sigma_{-c}; A_1', A_2'\big) - kV_i & \text{for claim } SC, \\ F\big((1-k)V_i, \sigma_{-c}; A_1', A_2'\big), & \text{other claims,} \end{cases} \tag{A.2}$$

where the limit is the left-hand-side or the right-hand-side limit, depending on whether V_i is the upper or the lower bound of the interval of stable regime. This results in an equation which is linear in the four unknowns A_1, A_2, A_1', A_2' and therefore allows eliminating one of these parameters.

(2) *Closure by bank management*. Suppose V_i is a trigger at which the bank's management decides to default, i.e., V_i is an absorbing barrier to the process V. Again, depending on the state (V, σ) of the bank, the market value of the claim prior to default can be written as $F(V, \sigma; A_1, A_2)$. Since the claim pays $\beta + \gamma V_i$ in case of closure, market equilibrium requires

$$\lim_{V \to V_i} F(V, \sigma; A_1, A_2) = \beta + \gamma V_i, \tag{A.3}$$

which eliminates one of the unknown parameters A_1, A_2.

(3) *Closure by regulators*. Suppose V_i is the bound of the closure region corresponding to the current asset volatility σ, i.e., $V_i = B(\sigma)$. In contrast to the boundaries discussed in the previous two points, V_i is not an absorbing barrier now, but instead the process V can freely enter and leave the closure region. Thus, according to the results of Feynman and Kac (see Björk, 1998, or on a more formal level, Karatzas and Shreve, 1988), market equilibrium requires that the value function of the claim is continuous and smooth at the boundary of the closure region,

$$\begin{aligned} \lim_{V \to V_i^-} F(V, \sigma; A_1, A_2) &= \lim_{V \to V_i^+} F(V, \sigma; A_1', A_2'), \\ \lim_{V \to V_i^-} F_V(V, \sigma; A_1, A_2) &= \lim_{V \to V_i^+} F_V(V, \sigma; A_1', A_2'). \end{aligned} \tag{A.4}$$

This condition yields two equations linear in A_1, A_2, A_1', A_2', eliminating two of these parameters.

(4) Suppose $V_i = c/r$ and the functional form of the claim's default payoff changes at c/r. Again, V_i is not an absorbing barrier, thus, boundary condition (A.4) has to be satisfied at c/r. Note, the functional form changes at c/r only for deposits and via (3) and (4) for deposit insurance and equity value, respectively. For charter value and switching costs condition (A.4) leads to $A_1 = A_1'$ and $A_2 = A_2'$.

(5) The last case we consider are boundary conditions for the situation where the interval of stable regime is unbounded—either from above or from below. Let $F(V, \sigma; A_1, A_2)$ denote the market value of the claim and, first, suppose $V_2 = \infty$, i.e., the interval of stability is unbounded from above. With higher asset values V, a switch of the regime of stability in the foreseeable future becomes less likely. Thus, for growing V the market value of

the claim has to converge to the market value of the constant profit flow α. Excluding speculative bubbles, we get boundary condition

$$\lim_{V\to\infty} F(V,\sigma;A_1,A_2)=\frac{\alpha}{r}. \tag{A.5}$$

Second, suppose $V_1=0$, i.e., the interval of stability is unbounded from below. Regarding that $V=0$ is a fixed point of the process (1), we can determine the market value of the claim at $V=0$. Market equilibrium for positive capital requirements ($B(\sigma)>0$) requires that

$$\lim_{V\to 0} F(V,\sigma;A_1,A_2)=\frac{\alpha}{r+\lambda}+\lambda\left(\frac{\beta}{r+\lambda}\right). \tag{A.6}$$

In both cases the respective boundary condition eliminates one of the unknowns A_1 and A_2.

Appendix B. Valuing a claim contingent on (V,σ)

B.1. The market value of deposits

As long as the bank is alive, depositholders receive a constant coupon flow c. In case of closure, the value of the claim is $\min\{V,c/r\}$. In terms of the general claim F (which we use in Section 2), the market value of deposits determines the parameters α, β and γ to

$$\alpha=c,\qquad \beta=\mathbf{1}_{[c/r,\infty)}c/r,\qquad \gamma=\mathbf{1}_{[0,c/r)}1. \tag{B.1}$$

The market value of debt in an interval of stable regime can be written as

$$D(V,\sigma;A_1,A_2)=\begin{cases}\dfrac{c}{r+\lambda}+\lambda\left(\mathbf{1}_{[c/r,\infty)}\dfrac{c/r}{r+\lambda}+\mathbf{1}_{[0,c/r)}\dfrac{1}{\lambda+\delta}V\right)\\ \qquad +A_1V^{x_1(\sigma)}+A_2V^{x_2(\sigma)}, \quad V\leqslant B(\sigma),\\ \dfrac{c}{r}+A_1V^{y_1(\sigma)}+A_2V^{y_2(\sigma)}, \quad V>B(\sigma).\end{cases} \tag{B.2}$$

The boundary conditions at the different bounds of stability are

- If (V_i,σ_c) is a switching threshold:

$$\lim_{V\to V_i} D(V,\sigma_c;A_1,A_2)=D\big((1-k)V_i,\sigma_{-c};A_1',A_2'\big). \tag{B.3}$$

- If V_i is a bankruptcy trigger:

$$\lim_{V\to V_i} D(V,\sigma;A_1,A_2)=\min\left\{V,\frac{c}{r}\right\}. \tag{B.4}$$

- If V_i is the bound of the closure region, i.e., $V_i=B(\sigma)$:

$$\begin{aligned}\lim_{V\to V_i^-} D(V,\sigma;A_1,A_2)&=\lim_{V\to V_i^+} D(V,\sigma;A_1',A_2'),\\ \lim_{V\to V_i^-} D_V(V,\sigma;A_1,A_2)&=\lim_{V\to V_i^+} D_V(V,\sigma;A_1',A_2').\end{aligned} \tag{B.5}$$

- If $V_i = c/r$:

$$\lim_{V\to V_i^-} D(V,\sigma;A_1,A_2) = \lim_{V\to V_i^+} D(V,\sigma;A_1',A_2'),$$
$$\lim_{V\to V_i^-} D_V(V,\sigma;A_1,A_2) = \lim_{V\to V_i^+} D_V(V,\sigma;A_1',A_2'). \tag{B.6}$$

- If the current regime is unbounded from above:

$$\lim_{V\to\infty} D(V,\sigma;A_1,A_2) = \begin{cases} \dfrac{c}{r+\lambda} + \lambda\left(\dfrac{c/r}{r+\lambda}\right), & B(\sigma)=\infty, \\ c/r, & B(\sigma)\neq\infty. \end{cases} \tag{B.7}$$

If it is unbounded from below:

$$\lim_{V\to 0} D(V,\sigma;A_1,A_2) = \begin{cases} c/(r+\lambda), & B(\sigma)\neq 0, \\ c/r, & B(\sigma)=0. \end{cases} \tag{B.8}$$

B.2. The value of charter value

Managing the asset portfolio, banks are able to generate excess cash flow (as motivated in Section 2). In the case of bankruptcy, the bank charter is irretrievably lost. Therefore, the parameters α, β, and γ which characterize this claim are

$$\alpha = \pi \in \{\pi_H, \pi_L\}, \qquad \beta = 0, \qquad \gamma = 0. \tag{B.9}$$

The present value of the bank charter in an interval of stable regime can be written as

$$CV(V,\sigma;A_1,A_2) = \begin{cases} \frac{\pi}{r+\lambda} + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)}, & V \leqslant B(\sigma), \\ \frac{\pi}{r} + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)}, & V > B(\sigma). \end{cases} \tag{B.10}$$

The boundary conditions at the different bounds of stability are

- If (V_i, σ_c) is a switching threshold:

$$\lim_{V\to V_i} CV(V,\sigma_c;A_1,A_2) = CV\big((1-k)V_i, \sigma_{-c}; A_1', A_2'\big). \tag{B.11}$$

- If V_i is a bankruptcy trigger:

$$\lim_{V\to V_i} CV(V,\sigma;A_1,A_2) = 0. \tag{B.12}$$

- If V_i is the bound of the closure region, i.e., $V_i = B(\sigma)$:

$$\lim_{V\to V_i^-} CV(V,\sigma;A_1,A_2) = \lim_{V\to V_i^+} CV(V,\sigma;A_1',A_2'), \tag{B.13}$$
$$\lim_{V\to V_i^-} CV_V(V,\sigma;A_1,A_2) = \lim_{V\to V_i^+} CV_V(V,\sigma;A_1',A_2').$$

- The value c/r does not change the functional form of the payoff one receives in case of closure. Thus, it is not a bound of stable regime.

- If the current regime is unbounded from above:

$$\lim_{V\to\infty} CV(V,\sigma;A_1,A_2) = \begin{cases} \pi/(r+\lambda), & B(\sigma)=\infty, \\ \pi/r, & B(\sigma)\neq\infty. \end{cases} \tag{B.14}$$

If it is unbounded from below:

$$\lim_{V\to 0} CV(V,\sigma;A_1,A_2) = \begin{cases} \pi/(r+\lambda), & B(\sigma)\neq 0, \\ \pi/r, & B(\sigma)=0. \end{cases} \tag{B.15}$$

B.3. The current value of switching costs

The claim SC denotes the current value of future switching costs, i.e., in the case of a switch at a threshold V_i, the immediate expenditure of kV_i is required. The remaining characteristics of this claim are

$$\alpha = 0, \qquad \beta = 0, \qquad \gamma = 0. \tag{B.16}$$

The market value of the switching-cost claim in an interval of stable regime can be written as

$$SC(V,\sigma;A_1,A_2) = \begin{cases} A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)}, & V \leqslant B(\sigma), \\ A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)}, & V > B(\sigma). \end{cases} \tag{B.17}$$

The boundary conditions at the different bounds of stability are

- If (V_i, σ_c) is a switching threshold:

$$\lim_{V\to V_i} SC(V,\sigma_c;A_1,A_2) = SC\big((1-k)V_i, \sigma_{-c}; A_1', A_2'\big) + kV_i. \tag{B.18}$$

- If V_i is a bankruptcy trigger:

$$\lim_{V\to V_i} SC(V,\sigma;A_1,A_2) = 0. \tag{B.19}$$

- If V_i is the bound of the closure region, i.e., $V_i = B(\sigma)$:

$$\begin{aligned} \lim_{V\to V_i^-} SC(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} SC(V,\sigma;A_1',A_2'), \\ \lim_{V\to V_i^-} SC_V(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} SC_V(V,\sigma;A_1',A_2'). \end{aligned} \tag{B.20}$$

- The value c/r does not change the functional form of the payoff one receives in case of closure. Thus, it is not a bound of stable regime.
- If the current regime is unbounded from above:

$$\lim_{V\to\infty} SC(V,\sigma;A_1,A_2) = 0. \tag{B.21}$$

If it is unbounded from below:

$$\lim_{V\to 0} SC(V,\sigma;A_1,A_2) = 0. \tag{B.22}$$

Appendix C. Determining the functional form of a claim

To show how the boundary conditions determine the constants A in the valuation equations for the claims involved in the analysis, we demonstrate one particular case. We explicitly derive the linear system that determines the value of debt D under the assumption that the critical thresholds are ordered in the manner: $B^* < S_H < c/r < B(\sigma_L) < B(\sigma_H) < S_L$. This assumption corresponds to the risk-shifting behavior illustrated in Fig. 2. The value function of debt is constructed by linking six functions of the form $D(V, \sigma, A_i, A_j)$ as defined in Eq. (B.2). As illustrated in Fig. C.1, each of these six functions is defined over an interval of stable regime and they are linked by the following boundary conditions:

At B^* the bank's equityholders default (while running the high-risk portfolio). The boundary condition follows from Eq. (B.4):

$$D(B^*, \sigma_H, A_1, A_2) = B^*. \tag{C.1}$$

At c/r the functional form of the valuation equation changes. Using Eq. (B.6), the corresponding boundary conditions for the low-risk bank are

$$\begin{aligned} &D(c/r, \sigma_H, A_1, A_2) = D(c/r, \sigma_H, A_3, A_4), \\ &D_V(c/r, \sigma_H, A_1, A_2) = D_V(c/r, \sigma_H, A_3, A_4). \end{aligned} \tag{C.2}$$

The threshold $B(\sigma_H)$ determines the border of the closure threshold of the high-risk bank. Using Eq. (B.5), the corresponding conditions are

$$\begin{aligned} &D\big(B(\sigma_H), \sigma_H, A_3, A_4\big) = D\big(B(\sigma_H), \sigma_H, A_5, A_6\big), \\ &D_V\big(B(\sigma_H), \sigma_H, A_3, A_4\big) = D_V\big(B(\sigma_H), \sigma_H, A_5, A_6\big). \end{aligned} \tag{C.3}$$

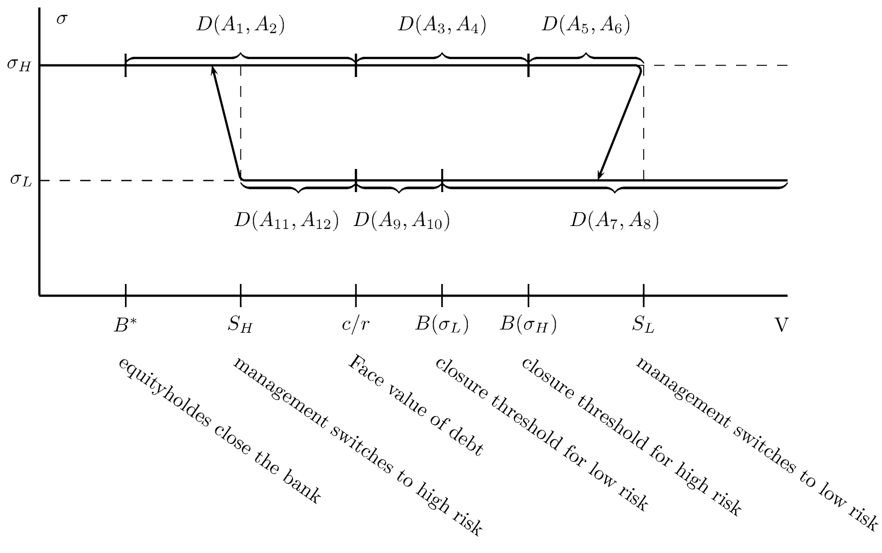

Fig. C.1. The value function of the banks assets consists of six functions that are defined over intervals of stable regime and linked by the respective boundary conditions.

At S_L the bank switches to low risk, and by using Eq. (B.3) we find that

$$D(S_L, \sigma_H, A_5, A_6) = D\big((1-k)S_L, \sigma_L, A_7, A_8\big). \tag{C.4}$$

The interval of stable regime for the low-risk bank is unbounded from above. Applying Eq. (B.7) yields

$$\lim_{V\to\infty} D(V, \sigma_L, A_7, A_8) = \frac{c}{r}. \tag{C.5}$$

Using Eq. (B.2) and the fact that $x_2(\sigma) > 0$, we can see that A_8 must equal zero.

The threshold $B(\sigma_L)$ determines the border of the closure threshold of the low-risk bank. Following Eq. (B.5), we find

$$\begin{aligned} &D\big(B(\sigma_L), \sigma_L, A_7, A_8\big) = D\big(B(\sigma_L), \sigma_L, A_9, A_{10}\big),\\ &D_V\big(B(\sigma_L), \sigma_L, A_7, A_8\big) = D_V\big(B(\sigma_L), \sigma_L, A_9, A_{10}\big). \end{aligned} \tag{C.6}$$

At c/r the functional form of the valuation equation changes again. The corresponding boundary conditions for the high-risk bank are

$$\begin{aligned} &D(c/r, \sigma_L, A_9, A_{10}) = D(c/r, \sigma_L, A_{11}, A_{12}),\\ &D_V(c/r, \sigma_L, A_9, A_{10}) = D_V(c/r, \sigma_L, A_{11}, A_{12}). \end{aligned} \tag{C.7}$$

At S_H the bank switches to high risk, therefore we require

$$D(S_H, \sigma_L, A_{11}, A_{12}) = D\big((1-k)S_H, \sigma_H, A_1, A_2\big). \tag{C.8}$$

Setting $A_8 = 0$, these equations define an 11-dimensional linear system

$$\begin{pmatrix} A_1\\ A_2\\ A_3\\ A_4\\ A_5\\ A_6\\ A_7\\ A_9\\ A_{10}\\ A_{11}\\ A_{12} \end{pmatrix} = M^{-1} \begin{pmatrix} B^* - \frac{B^*\lambda}{\delta+\lambda} - \frac{c}{\lambda+r}\\ -\frac{c\lambda}{(\delta+\lambda)r} + \frac{c\lambda}{r(\lambda+r)}\\ -\frac{\lambda}{\delta+\lambda}\\ \frac{c}{r} - \frac{c}{\lambda+r} - \frac{c\lambda}{r(\lambda+r)}\\ 0\\ 0\\ -\frac{c}{r} + \frac{c}{\lambda+r} + \frac{c\lambda}{r(\lambda+r)}\\ 0\\ \frac{c\lambda}{(\delta+\lambda)r} - \frac{c\lambda}{r(\lambda+r)}\\ \frac{\lambda}{\delta+\lambda}\\ \frac{\lambda S_H}{\delta+\lambda} - \frac{(1-k)\lambda S_H}{\delta+\lambda} \end{pmatrix}, \tag{C.9}$$

where the matrix M is defined as

$$M = \left[\begin{array}{ccccccccccc}
B^{*x_1(\sigma_H)} & B^{*x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\left(\frac{c}{r}\right)^{x_1(\sigma_H)} & \left(\frac{c}{r}\right)^{x_2(\sigma_H)} & -\left(\frac{c}{r}\right)^{x_1(\sigma_H)} & -\left(\frac{c}{r}\right)^{x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x_1(\sigma_H)\left(\frac{c}{r}\right)^{x_1(\sigma_H)-1} & x_2(\sigma_H)\left(\frac{c}{r}\right)^{x_2(\sigma_H)-1} & -x_1(\sigma_H)\left(\frac{c}{r}\right)^{x_1(\sigma_H)-1} & -x_2(\sigma_H)\left(\frac{c}{r}\right)^{x_2(\sigma_H)-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & B(\sigma_H)^{x_1(\sigma_H)} & B(\sigma_H)^{x_2(\sigma_H)} & -B(\sigma_H)^{y_1(\sigma_H)} & -B(\sigma_H)^{y_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x_1(\sigma_H)B(\sigma_H)^{x_1(\sigma_H)-1} & x_2(\sigma_H)B(\sigma_H)^{x_2(\sigma_H)-1} & -y_1(\sigma_H)B(\sigma_H)^{y_1(\sigma_H)-1} & -y_2(\sigma_H)-1B(\sigma_H)^{y_2(\sigma_H)-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & S_L^{y_1(\sigma_H)} & S_L^{y_2(\sigma_H)} & -((1-k)S_L)^{y_1(\sigma_L)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & B(\sigma_L)^{y_1(\sigma_L)} & -B(\sigma_L)^{x_1(\sigma_L)} & -B(\sigma_L)^{x_2(\sigma_L)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_1(\sigma_L)B(\sigma_L)^{y_1(\sigma_L)-1} & -x_1(\sigma_L)B(\sigma_L)^{x_1(\sigma_L)-1} & -x_2(\sigma_L)B(\sigma_L)^{x_2(\sigma_L)-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{c}{r}\right)^{x_1(\sigma_L)} & \left(\frac{c}{r}\right)^{x_2(\sigma_L)} & -\left(\frac{c}{r}\right)^{x_1(\sigma_L)} & -\left(\frac{c}{r}\right)^{x_2(\sigma_L)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1(\sigma_L)\left(\frac{c}{r}\right)^{x_1(\sigma_L)-1} & x_2(\sigma_L)\left(\frac{c}{r}\right)^{x_2(\sigma_L)-1} & -x_1(\sigma_L)\left(\frac{c}{r}\right)^{x_1(\sigma_L)-1} & -x_2(\sigma_L)\left(\frac{c}{r}\right)^{x_2(\sigma_L)-1} \\
((1-k)S_H)^{x_1(\sigma_H)} & ((1-k)S_H)^{x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_H^{x_1(\sigma_L)} & -S_H^{x_2(\sigma_L)}
\end{array}\right]. \tag{C.10}$$

If the critical thresholds (B^*, S_H, c/r, $B(\sigma_L)$, $B(\sigma_H)$, S_L) are ordered in a different way, a similar procedure has to be applied. The solution of the other claims (*CV*, *DI*, *SC*, and *E*) is analogous. For given B^*, S_H, S_L, all value functions are well-defined. These managerial decision variables are determined numerically with the objective of maximizing the value of equity (see Section 3.1).

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