Risk-Minimizing Equity Strategies

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Institutional asset management in a low-interest rate environment

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Background: Post-2008 crisis

- Central banks in the U.S. and Europe have pursued an expansionary monetary policy (quantitative easing)
- Interest rates on government debt at historic lows
- Consequently, real interest rates are zero or negative
- Pressure on investment-grade bonds has led to yield compression
- New solvency regulations limit investment in higher-risk assets

Current market conditions: Still unfavorable

- Large fiscal deficit and debt in United States and Europe
- Slowdown in China and Japan (though, India is picking up)
- Deteriorating conditions in the Middle East
- Looming threat of inflation

How should we respond?

- One response: Tell our clients to
  - "face up to reality."
- A second response: Tell our clients that
  - "inflation is also low; therefore, real returns are not so bad."
- A third response: Take on more risk

To improve return, one can increase ...

- Leverage (if possible)
- Duration risk (but beware: interest rates can only go up)
- Credit risk
- Currency risk
- Inflation risk
- Liquidity risk
- Emerging-market risk
- Equity-market (beta) risk

When taking on more risk . . .

- Critical to mange the risk of the portfolio.
- How best to do this?
  - What are the various risk-minimizing (equity) strategies?
  - How to choose amongst these strategies?

Insights of Classical Mean-Variance Portfolios

- It is optimal to hold Markowitz frontier portfolios;
  A large asset base improves portfolio frontier — diversification.
- An individual asset's risk is its contribution to portfolio risk.
  - **Variance** of each asset contributes little to portfolio risk;
  - **Covariances** between assets determine portfolio risk.
- Theory is sensible, logical, and sophisticated.

Performance of Mean-Variance Portfolios

- The inputs for determining mean-variance optimal portfolios are the expected-returns vector and variance-covariance matrix.
  - One can improve the precision of the estimate of the variance-covariance matrix by using higher-frequency data.
  - The precision of the estimate of expected returns depends
    - only on the length of the data series, and
    - so cannot be improved by using higher-frequency data.
- Out-of-sample performance of Markowitz mean-variance optimal portfolios that ignore estimation error is very poor.

Alternative-weighted portfolios

- One can consider three types of portfolios that do not rely on estimates of mean returns:

1. **Fundamental-weighted portfolios:**
   - These portfolios are based on fundamentals such as sales, dividends, earnings, etc., rather than financial measures of size.
2. **Minimum-variance portfolios:**
   - The optimal weights are based only on risk estimates.
   - Norm constraints improve out-of-sample performance further (DeMiguel, Nogales, and Uppal (2014))
   - Dangl and Kashofer (2013) provides an excellent discussion of the performance of these portfolios over time.
3. **Risk-parity portfolios:**
   - These portfolios are based only on risk estimates but there is no optimization.

Risk-Budgeting Portfolios: Characteristics

- These portfolios do not rely on estimates of expected returns.
- They do not use optimization.
- They lead to portfolios that are diversified (that is, the weights are not concentrated).

Risk-Budgeting Portfolio: Definition – Dollars

- The risk contribution of an asset i, denoted by $\mathcal{RC}_i$, is given by:

$$\mathcal{RC}_i = [\text{Weight of } i] \times [\text{Marginal contribution of } i \text{ to portfolio risk}]$$

$$= \mathrm{w}_i \times \frac{\sigma_{ip}}{\sigma_p} = \mathrm{w}_i \, \frac{\Sigma\, \mathrm{w}}{\sqrt{\mathrm{w}^\top \Sigma\, \mathrm{w}}}. \tag{1}$$

- Consider the dollar risk-budgets for the N assets in a portfolio:

$$\{[\text{Risk budget}]_1, [\text{Risk budget}]_2, \ldots, [\text{Risk budget}]_N, \}$$

- A risk budgeting portfolio w is then defined as one that satisfies the following set of risk-budgeting constraints:

$$\begin{aligned} \mathcal{RC}_1 &= [\text{Dollar risk budget}]_1 \\ \mathcal{RC}_2 &= [\text{Dollar risk budget}]_2 \\ \cdots &= \cdots \\ \mathcal{RC}_N &= [\text{Dollar risk budget}]_N \end{aligned}$$

Risk-Budgeting Portfolio: Definition – Percentages

- Consider a set of percentage risk-budgets for the N assets in a portfolio: $\{b_1, b_2, \ldots, b_N\}$.
- A risk-budgeting portfolio w is defined as one that satisfies the following system of equations:

$$\mathcal{RC}_i = [\text{Percentage risk budget}]_i \times [\text{Risk of total portfolio}]$$

$$= b_i \, \sigma_p \quad \ldots \text{ risk-budgeting constraint} \tag{2}$$

$$\mathrm{w}_i \geq 0 \quad \ldots \text{ each portfolio weight is non-negative} \tag{3}$$

$$b_i > 0 \quad \ldots \text{ each percentage risk budget is positive} \tag{4}$$

$$\sum_{i=1}^{N} b_i = 1 \quad \ldots \text{ the percentage risk budgets add up to 1} \tag{5}$$

$$\sum_{i=1}^{N} \mathrm{w}_i = 1 \quad \ldots \text{ the portfolio weights add up to 1.} \tag{6}$$

Interpretation of Risk-Budgeting Portfolio Weights

- For expositional ease, assume only two uncorrelated assets:

$$w_1 = \frac{\frac{\sqrt{b_1}}{\sigma_1}}{\frac{\sqrt{b_1}}{\sigma_1} + \frac{\sqrt{b_2}}{\sigma_2}} \quad \text{. . . increasing in sq. root of risk budget, decreasing in volatility}$$

$$= \frac{\frac{b_1}{\beta_1}}{\frac{b_1}{\beta_1} + \frac{b_2}{\beta_2}} \quad \text{. . . increasing in its risk budget, decreasing in beta.}$$

- Risk-budgeting portfolios can be interpreted as minimum-risk portfolios subject to a constraint on portfolio diversification.
  - In this case, just like in Jagannathan and Ma (2003), one can show that the risk-budgeting portfolio is a minimum-variance portfolio with shrinkage of the covariance matrix.

Equal-Risk-Contribution Portfolio: Definition

- A special case of the risk-budgeting portfolio is the equal-risk-contribution portfolio, where all the risk budgets are equal:

$$b_i = b_j = \frac{1}{N}.$$

- Thus, the equal-risk-contribution portfolio inherits all the properties of the risk-budgeting portfolio.
- The equal-risk contribution portfolio also has some additional properties derived by Maillard, Roncalli, and Teiletche (2010):

$$\mathrm{w}_1 = \frac{\frac{1}{\sigma_1}}{\frac{1}{\sigma_1} + \frac{1}{\sigma_2}},$$

implying that the weight of an asset is

  - inversely proportional to its volatility, and
  - independent of correlation.

Understanding Portfolio Weights From Different Criteria

- It is difficult to decide how best to choose a portfolio that is "optimally diversified".

Portfolio	Evaluation in terms of "optimal divers."
• Mean-variance optimal portfolio	Portfolio on the efficient frontier
• Minimum-variance optimal portfolio	Portfolio on the efficient frontier
• Equal-weighted portfolio	Has the lowest weight concentration
• Weight-budgeting portfolio	Arbitrary weight budgets
• Equal-risk-contribution portfolio	Lowest risk concentration
• Risk-budgeting portfolio	Arbitrary risk budgets

Comparing Different Portfolio Strategies

1. Equal-weight portfolio: $\mathsf{w}_i = \mathsf{w}_j$
2. Weight-budgeting portfolio: $\dfrac{\mathsf{w}_i}{b_i} = \dfrac{\mathsf{w}_j}{b_j}$
3. Minimum-variance portfolio: $\dfrac{\partial \sigma_p}{\partial \mathsf{w}_i} = \dfrac{\partial \sigma_p}{\partial \mathsf{w}_j}$
4. Risk-budgeting portfolio: $\dfrac{\mathcal{RC}_i}{b_i} = \dfrac{\mathcal{RC}_j}{b_j}$
5. Equal-risk-contribution portfolio: $\mathcal{RC}_i = \mathcal{RC}_j$

Risk-Budgeting Portfolios: Performance

- These portfolios outperform cap-weighted portfolio.
- These portfolios have exposure to factors other than value and size, such as low idiosyncratic volatility, low beta, momentum.
- Risk-budgeting portfolios may provide
  - better conditional performance across bull/bear markets;
  - more control on factor exposure;
  - better diversification, especially when strategies are combined.

Risk-Budgeting Portfolios: Factor Exposures

- Performance of smart beta strategies is not fully explained by value and small cap exposures.
  - Frazzini and Pedersen (2014), Asness, Moskowitz, and Pedersen (2013), and Asness, Frazzini, and Pedersen (2013) provide empirical evidence that stock portfolios seeking exposures to low beta, momentum or quality factors generate returns that cannot be explained by the Fama French factors.
    - Spängler IQAM has a very successful quality equity fund, the "Spängler IQAM Quality Equity Europe"
  - Amenc, Goltz, and Lodh (2014) argue that many smart beta strategies seek factor exposures other than value and small cap, for example momentum or low risk.

Risk-Budgeting Portfolios: Conclusion I

Thiagarajan and Schachter (2011) summarize risk-parity portfolios:

- The attractiveness of risk-parity portfolios is not that it outperforms mean-variance or market-capitalization portfolio.
- The attractiveness of risk-parity is not its simplicity.
  - The equal-weighted strategy is far simpler to implement
- However, risk-parity portfolios strongly appeal to our intuition that risk diversification is central goal in portfolio selection
- Risk-parity portfolios are appealing also because they do not depend on expected returns.

Risk-Budgeting Portfolios: Conclusion II

- There are several benefits of risk-budgeting portfolios:

  1. Risk-budgeting portfolios outperform cap-weighted portfolio.
  2. Risk-based strategies take on exposures to other factors than value and small cap, such as
     - low beta,
     - low idiosyncratic volatility, and
     - momentum.
  3. Selecting stocks by their characteristics allows one to manage the factor tilts of diversification-based weighting schemes.
  4. These strategies can be combined to diversify strategy-specific risks.

Thank you!

Suggestions for Further Reading I

- Risk-Budgeting and Equal-Risk-Contribution Portfolios
  - Bhansali (2012)
  - Bruder and Roncalli (2012)
  - Chaves, Hsu, Li, and Shakernia (2011, 2012)
  - Demey, Maillard, and Roncalli (2010)
  - Hereil and Roncalli (2011)
  - Inker (2011)
  - Lee (2011)
  - Lohre, Neugebauer, and Zimmer (2012)
  - Maillard, Roncalli, and Teiletche (2010)

Suggestions for Further Reading II

- Peters (2011)
- Qian (2005, 2011, 2012)
- Roncalli (2013) — excellent book, and good starting point for learning about risk-budgeting portfolios
- Sebastian (2012)
- Thiagarajan and Schachter (2011)
- Ruban and Melas (2011).

- Most-Diversified Portfolios
  - Choueifaty and Coignard (2008)
  - Choueifaty, Froidure, and Reynier (2011)

Suggestions for Further Reading III

- Volatility-Based Maximum Sharpe Ratio Portfolios
  - Amenc, Goltz, Martellini, and Retkowsky (2010)
  - Martellini (2008)
- Risk Parity Portfolios with Risk Factors
  - Deguest, Martellini, and Meucci (2013)
  - Lohre, Neugebauer, and Zimmer (2012)
  - Meucci (2009)
  - Roncalli (2013)
  - Roncalli and Weisang (2012)

Suggestions for Further Reading IV

- Performance Evaluation of Alternative-Weighted Portfolios
  - Amenc (2011)
  - Amenc, Goltz, and Martellini (2011a,b)
  - Amenc, Goltz, and Tang (2011)
  - Amenc, Goltz, and Lodh (2012,?)
  - Amenc, Goltz, Lodh, and Martellini (2012),
  - Amenc, Goltz, Martellini, and Retkowsky (2010)
  - Amenc, Goltz, and Martellini (2013)
  - Chow, Hsu, Kalesnik, and Little (2011)
  - De Carvalho, Lu, and Moulin (2012)

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