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Optimal capital allocation using RAROC™ and EVA® ☆

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Text

Abstract

Equity capital allocation plays a particularly important role for financial institutions such as banks, who issue equity infrequently but have continuous access to debt capital. In such a context this paper shows that EVA and RAROC based capital budgeting mechanisms have economic foundations. We derive optimal capital allocation under asymmetric information and in the presence of outside managerial opportunities for an institution with a risky and a riskless division. It is shown that the results extend in a consistent manner to the multidivisional case of decentralized investment decisions with a suitable redefinition of economic capital. The decentralization leads to a charge for economic capital based on the division's own realized risk. Outside managerial opportunities increase the usage of capital and lead to overinvestment in risky projects; at the same time more capital is raised but risk limits are binding in more states. An institution with a single risky division should base its hurdle rate for capital allocated on the cost of debt. In contrast, the hurdle rate tends to the cost of equity for a diversified multidivisional firm. The analysis shows that hurdle rates have a common component in contrast to the standard perfect markets result with division-specific hurdle rates.

Keywords: Capital; Allocation; EVA; RAROC; Banking; Risk; Value at risk; Equity; Financial institutions; Divisional; Multidivisional; Limits; Hurdle rate; Cost of equity; Overinvestment; Underinvestment; Incentives; Limits; Diversification; Outside options; Mechanism; Economic capital

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1. Introduction

In frictionless markets without asymmetric information the theory of corporate finance provides clearly specified rules for firms' optimal capital budgeting decisions. In such an environment capital is flexible and management simply distributes funds to the divisions as if they were autonomous entities. This paradigm contrasts sharply with the observed budgeting procedures and incentive schemes actually utilized by firms, especially those with multiple divisions. A critical ingredient of these procedures involves the allocation of capital from the central management to the business units.

Capital allocation plays a particularly important role for financial institutions such as banks, who issue equity infrequently but have continuous access to debt capital. These firms typically face frictions of the following sort:

(1) debt is favored over equity up to some limit based on the riskiness of the underlying assets;
(2) equity capital must be raised in advance of precise information about investment opportunities; and
(3) regulations link their risk-taking ability to equity capital.

Banks, by their very nature, are in the business of accepting deposits, which implies a bias in favor of debt financing. The provision of services such as liquidity through demand deposits, letters of credit etc. imply that depositors are willing to loan their funds at rates below those prevailing in money markets. Thus, banks will prefer high leverage. Regulations, such as those promulgated by the Basle Committee, prescribe minimum equity capital based on the risk of the bank's assets, which can change quickly due to asset allocation decisions and the volatility of asset markets.

In this setting, banks and other financial institutions have sought to base their capital allocation processes on shareholder value concepts such as Risk Adjusted Return on Capital (RAROC) and Economic Value Added (EVA) in recent years.[1] Some of the motivation for these approaches has come from the initiatives of the Basle Banking Committee in defining international capital requirements. There is a variety of versions of these concepts that have been adopted, and the academic literature has provided limited guidance on the optimal form of such capital allocation mechanisms, especially when there are multiple divisions subject to agency problems of asymmetric information. The purpose of our paper is to show that EVA and RAROC based performance evaluation and reward schemes have economic foundations in a theoretical model of capital budgeting. Furthermore, we show that the analysis at the level of a single division adapts itself in a consistent manner to the general multidivisional case of decentralized investment decisions with suitable redefinition of economic capital. We focus on how much capital should be allocated to a division with a particular reported investment opportunity, what its hurdle rate should be and how much equity capital the institution should raise *ex ante* before the investment opportunities become known.

The optimal capital allocation mechanism we derive requires the institution to compute *economic capital*, rather than a book capital for use in the EVA and RAROC computations. In practice the issue of whether economic capital should be based on risk use or risk limits is a subject of considerable discussion. We find that economic capital must be proportional to the

[1] Uyemura et al. (1996) discusses the use of these concepts in general. James (1996) and Zaik et al. (1996) describe the use of these techniques at Bank of America.

amount of risk actually utilized, rather than some ex ante determined amount that corresponds to the division's risk limit. As a result, divisional managers do not always fully utilize their position limits. They voluntarily curtail their risk taking activities in some cases leaving excess equity capital invested in less risky but also less productive investments. These inefficiencies are sometimes necessary in order to elucidate truthful information across divisions.

In a model where there is one division with risky opportunities and another division which invests in riskless assets such as treasuries, we find that the optimal mechanism imposes a strict risk limit, coupled with a capital charge based on divisional risk actually chosen. The optimal hurdle rate is based on the cost of debt but can be either greater or less depending on the type and severity of the asymmetric information problem. The key determining factor on whether there is over- or underinvestment is whether divisional management has transferrable benefits outside the firm. Furthermore we find that a financial institution should raise more equity as the future uncertainty increases, thus giving managers the freedom to take advantage of their real investment opportunities. Therefore risk limits are binding in fewer states as the uncertainty about future investment opportunities increases.

These results are extended to a general n-divisional firm with multiple risky divisions. An important result here is that the economic capital charged to a division is made up of a coefficient times the own (specific) risk of that division. This implies that the capital allocation mechanism can be decentralized to the local level. The coefficient takes care of the impact of a divisions own investment activities in relation to those of the overall institution. We show that in the limit as the number of risky divisions increases without bound, this coefficient is related to the amount of systematic risk of the institution whereas for a small number of divisions, the coefficient that determines the capital charge also depends on divisions' unsystematic risk.

The RAROC hurdle rate in the n-divisional case is also derived. We find that the hurdle rate is normally higher than in the case when there is a single risky division. This is because the opportunity cost of investment now is the expected return to be earned in other risky investment activities within the institution, which converges to the cost of equity in the limit when the number of divisions goes to infinity. Therefore in the multiple divisional firm, both the return to economic capital is greater (because the economic capital is lower) as well as the hurdle rate than in a single division firm. Once again EVA and RAROC can be suitably employed to provide optimal incentives for designing the portfolio of investment activities.

The influence of asymmetric information at the divisional level depends on the extent of outside opportunities the private information brings to the manager. When outside options are limited, as may be the case, for instance, in a mortgage lending division, the mechanism attempts to appropriate the informational advantage of divisional managers. As a result, the hurdle rate is raised to induce underinvestment, thereby making underreporting profit opportunities more costly. On the other hand for a trading division, where information may be freely transferrable outside the institution, the optimal mechanism provide for overinvestment, so that managers with unfavorable information are dissuaded from overreporting their true investment opportunities. Therefore what appears to be excessive risk-taking for such divisions may actually be part of an optimal response to the implicit contracting problem of retaining managerial talent.

Froot and Stein (1998) discuss the problem of divisional interdependence in a model in which risk management arises endogenously from the need to avoid an adverse selection problem with respect to external finance. We extend this research by showing that the conditions under which EVA and RAROC can be justifiably employed include those where divisional management is privately informed about their own investment opportunities and exercises local control.[2] Cuoco and Liu (2006) analyze a dynamic portfolio problem under a value at risk constraint and discuss the conditions required to get an institution to truthfully report their risk levels. In contrast, we assume that *ex post* risk is observable and utilized as part of the optimal mechanism. Perold (2005) discusses some of the popular approaches to multidivisional capital allocation and the issues that arise. The method we propose involves the central authority of the institution specifying a mechanism under which divisions are charged an internal price for economic capital.[3] In some ways our results are therefore reminiscent of the literature on internal capital markets (Stein, 1997). However an important distinction is that the "price" must be personalized for the division's own investment opportunities. The use of this risk pricing mechanism "separates" the investment decisions in a way that allows each division to act independently of the others.

As in other models of capital budgeting under asymmetric information, we derive distortions relative to *first best* investment policies. One of the original papers to look at capital budgeting under asymmetric information and solve for the amount of inefficiency induced was Harris et al. (1982). Harris and Raviv (1996, 1998) analyze capital budgeting decisions in the presence of asymmetric information about project quality and empire building preferences by divisional managers. At a cost, headquarters can obtain information about a division's investment opportunity set. The paper demonstrates under which circumstances headquarters will delegate the decision how to allocate capital across projects and what form this delegation may take. In these papers, distortions can either be in the form of under or overinvestment. Bernardo et al. (2001, 2004) discuss this issue using optimal compensation for the manager and argue that underinvestment will always prevail, because now the compensation contract can be designed to extract surplus. By contrast, in our model we specialize the situation to the setting of financial institutions where capital is related to risk and raised ex ante before being allocated. We generalize the extent of managerial reservation utilities due to outside opportunities and identify overinvestment with increasing outside opportunities.[4]

The remainder of the paper is structured as follows. Section 2 discusses the economic and institutional environment and develops the model. Section 3 provides the analysis for the case of an institution optimally allocating capital between riskless investments and a single risky division. Section 4 shows how the optimal incentive mechanisms and hurdle rates are determined. Comparative statics in the context of an illustrative example are considered in Section 5. Section 6 extends the model to the case of multiple risky divisions. Section 7 concludes.

2. Model development

Financial institutions and banks, in particular, face market imperfections such as costs of financial distress, transactions costs in accessing capital markets, or simply regulatory constraints. These frictions imply that risk management, capital structure and capital budgeting are interde-

[2] Ad hoc procedures for capital allocation, such as those in Kimball (1997) may create significant distortions in the decisions taken.

[3] Throughout this paper we assume that divisions are all financed by the same corporate entity. This is in contrast to Kahn and Winton (2004) where the firm can separate the financing of the divisions.

[4] Milbourn and Thakor (1996) consider an asymmetric information model of capital allocation and compensation. They focus on the moral hazard problem of managerial effort as well as enhanced bargaining opportunities for the central authority.

pendent. We begin by discussing the nature of investment opportunities faced by the financial institution.

2.1. Investment opportunities

We first specify how a business unit's investment opportunities are modeled. A financial institution consists of n divisions, each of which may choose investment projects. The cash flow of division, i, net of directly attributable (non-financing) costs are defined by

$$\pi_i = \mu_i(\sigma_i)\theta_i + \sigma_i z_i, \tag{1}$$

where σ_i represents the investment decision in terms of the standard deviation of risk taken, $\mu_i(\sigma_i)\theta_i$ represents expected cash flows conditioned on an information variable, θ_i and z_i is a zero mean, unit standard deviation normally distributed random variable. The interpretation of θ_i is that it represents an index of the favorability of investment opportunities. A higher θ_i makes risk more productive and makes a higher σ_i more desirable.

We make some further assumptions about the functional form of the relation between risk and cash flows. We assume that, *ceteris paribus*, more "aggressive" risk taking by a division translates into higher expected returns, i.e.

$$\frac{\partial \mu_i}{\partial \sigma_i} \equiv \mu_{i\sigma} > 0. \tag{2}$$

We also assume that the investment technology is concave, $\partial^2 \mu_i / \partial \sigma_i^2 < 0$, and that marginal returns go to zero as risk increases, $\lim_{\sigma_i \to \infty} \partial \mu_i / \partial \sigma_i = 0$.

We make the additional expositional simplification that the investment activity of division i requires a total financing requirement of $A_i \sigma_i$ dollars at the initial time period, where $A_i \geqslant 0$ is a constant coefficient for division i representing the amount of *physical* investment capital required. Depending on the nature of activities for each division, this coefficient could be very different. For instance lending requires substantial financing requirements. On the other hand, for certain derivatives trading such as swaps the funding requirements are virtually zero, in principle. The assumption of proportionality between physical capital and cash flow risk is commonly employed when the investment involves a holding in frictionless capital markets, such as in the case of equity or bond trading. In such cases, the constant A_i is equal to the reciprocal of the standard deviation in rate of return units.[5]

The relation $\mu_i(\sigma_i)$ is a reduced form, representing the composition of a number of concurrent operating activities of financial institutions. For instance consider a division engaged in deposit-financed lending activity. Let L denote the amount of lending undertaken and assume that the marginal loan quality is decreasing in the aggregate amount of lending and eventually drops below the cost of deposit funds. We can then represent the expected net cash flows from lending activities by $\mu = f(L)$ with f being an increasing concave function. Suppose, for instance, that the risk of the loan portfolio is increasing linearly in total lending, so that $\sigma = sL$ with $s > 0$. In this case expected cash flows can be described by

$$\mu_i(\sigma_i) \equiv f\left(\frac{\sigma_i}{s}\right).$$

[5] For example, if the standard deviation of equity returns is 0.20, the coefficient would be $A_i = 1/0.2 = 5$.

2.2. Capital

We now specify the objective function of the financial institution. The institution is being run in an environment where shareholders interests are paramount and so wishes to maximize shareholder value subject to any constraints given by the regulatory environment. The institution first raises equity capital, C, at an exogenously determined cost of capital, r_E. In keeping with capital market theory, the firm participates in competitive capital markets and therefore takes the cost of equity as given. Of course this cost of capital should reflect the business risks of the financial institution as well its leverage. Since we will later assume that leverage at the institutional level is regulated, and that all institutions will be constrained in the same way, we assume that r_E is constant.[6] The second source of funds is debt or deposits, the cost denoted by r_D, which we also assume to be constant. Summing the cash flows over all divisions less the costs associated with debt financing, gives the cash flows attributable to equity capital, or net income. This net income minus the cost for equity capital is defined as EVA, in accordance with the modern perspective of shareholder value. Taking expectations, expected EVA is denoted by

$$EVA = \sum_i \mu_i(\sigma_i)\theta_i - r_D\left(\sum_i A_i\sigma_i - C\right) - r_E C. \tag{3}$$

That is, the total financing requirement is $\sum_i A_i\sigma_i$, out of which C is made up of equity and the rest is debt financed. EVA represents the annual contribution to shareholder value. It is well known that the net present value (NPV) equals the discounted sum of EVAs.[7]

Of course, if there are no capital market imperfections facing a financial institution, the Modigliani and Miller Theorem would apply and the problem of capital allocation is non-existent. We specify two types of imperfections faced by a bank or financial institution. The first imperfection implies that the financial institution wants to minimize its use of equity capital. This occurs for a number of reasons. First, due to liquidity and security offered to depositors, bank deposits are considered to be a cheap source of capital. Second, as in other industries, debt offers a tax subsidy due to deductibility of interest. Third, as emphasized by Merton and Perold (1993), a bank's customers are frequently also its debtholders so that the nature of banking requires leverage. The more "customers" a bank wants to attract, the more debt it must be prepared to accept in its capital structure. In our model, we capture these effects by assuming that the (after-tax) cost of deposits, r_D, is less than the cost of equity, r_E.

The second type of imperfection occurs when banks cannot raise equity capital instantaneously after newly acquired information. Accordingly we assume that the firm must raise equity capital before learning divisional-specific information about investment opportunities. If the equity capital raised exceeds the investment opportunities, then it is invested in a riskless asset, paying the same rate of return, r_D, as the cost of debt. The fact that this "extra" equity is invested with a lower rate of return than its cost essentially means that the financial institution incurs deadweight costs which, as in Froot and Stein (1998) may be interpreted as a tax.[8]

[6] See Myers and Majluf (1984) for a well known model in which asymmetric information generates a cost to raising equity capital and therefore causes the firm to utilize debt financing before resorting to equity.

[7] We use the term EVA following the terminology of Stern Stewart who popularized it, although many academic researchers refer to this as *residual income*. Expressing NPV as the sum of discounted EVAs is known in Accounting circles as the Edwards–Bell–Ohlson model after Edwards and Bell (1961) and Ohlson (1995).

[8] See Perold (2005) for a further discussion of the importance and examples of deadweight costs in the context of financial institutions.

Depositors and other debtholders are only willing to lend their capital to a bank if it has a sufficient amount of equity capital to ensure its solvency. In addition regulators specify minimum equity standards which are based on various risk measures. We focus in this paper on the use of the most popular of these standards, namely that of *value at risk* (VaR) to establish equity capital requirements. We assume therefore that the equity capital of the bank, C, must satisfy the constraint

$$C \geqslant \mathrm{VaR} = \alpha\sigma, \tag{4}$$

for normally distributed cash flows. According to Eq. (4), a bank is limited in its risk taking such that the resulting VaR does not exceed the amount of equity capital. This capital structure constraint ultimately determines the required capital allocation to investment projects or divisions.[9]

2.3. The agency problem

In our model the divisions of the financial institution are run by individual managers who can obtain better information about investment opportunities than the top management, by observing the parameter θ_i. To simplify the analytics we assume they are risk neutral, although they will be charged an amount for the risk they impose on the firm. In such a setting it has been demonstrated that optimal compensation contracts are linear in terms of ex post realized cash flows (McAfee and McMillan, 1986) and (Laffont and Tirole, 1987). These papers show that the part of the contract contingent on the observed outcome affects effort expenditure. Although we do not formally model effort expenditure, we rely on the idea that the division manager must be given some positive fraction, γ_i, of the cash flows in order to be motivated to become informed.[10]

We specify the compensation given to the division manager as

$$\phi_i = \gamma_i \pi_i - T_i. \tag{5}$$

The key question addressed in this paper is the optimal functional form of T_i, which we interpret as a capital charge assessed against the managers share of divisional cash flows. The design of the *capital allocation* system must induce the manager to select investments generating the optimal institutional level of risk. We will show how this can be interpreted as the use of divisional EVA and RAROC.

Taking expectations the utility of the manager is then expressed as

$$U_i = E[\phi_i|\theta_i] = \gamma_i \mu_i(\sigma_i)\theta_i - T_i. \tag{6}$$

Since the capital allocation and optimal compensation are being chosen by the institution, it is necessary to specify some *reservation*, or default utility that the manager would otherwise be able to obtain. It is of fundamental importance how this utility varies with the private information of the manager. We specify this reservation utility as

$$U_i(\theta_i) \geqslant \underline{U} + \eta_i \mu_i(\sigma_i)\theta_i. \tag{7}$$

[9] The Basle Accord allows banks to use their own internal models to calculate equity requirements. In this case a standard of 99% for ten days times a multiplier of between 3 and 4 is used for the bank's trading activities. That is, α is between 6.9 and 9.2, and σ is defined as the ten day standard deviation.

[10] Although we know that the optimal contract will involve a positive γ_i, our paper is not focused on solving for it. In general the level is typically determined by the ratio of expected cash flows to the unobservable costs of effort and is inversely related to this ratio. We have specified a numerical example similar to Section 5, available on request, with unobservable effort and solved for the optimal value of γ_i. The optimal value for γ_i was bounded away from zero.

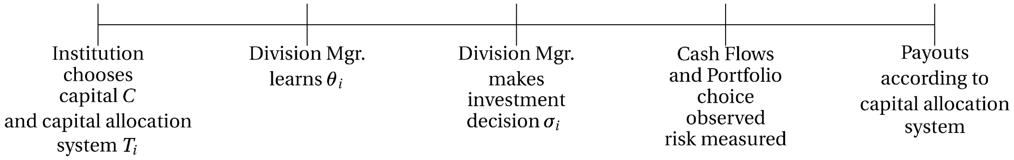


Fig. 1. Sequence of events.

We consider two polar situations. The first, $\eta_i = 0$, is where the manager has very limited bargaining power, in that his *outside options* are not a function of private information. This represents an economic situation in which the information might be thought of as specific to the parent institution and not transferable. The second situation, $\eta_i > \gamma_i$, holds when the manager has outside options which are more significant than the incentives generated by the firm internally via the fraction of the cash flows paid to the manager, γ_i.[11] An example for such a situation may be a manager who is managing an equity portfolio, and who contemplates setting up his own hedge fund where he is the sole owner. In this case η_i would be equal to one and $-\underline{U}$ would be the fixed cost of setting up the firm. Considering only the two polar cases $\eta_i = 0$ and $\eta_i > \gamma_i$ greatly simplifies the mathematical analysis of asymmetric information in the next section.

The first step in the institution's risk management problem is to select the amount of equity capital, C, to raise. Then the institution elicits information from each of the managers about their investment opportunities based on the commitment to a mechanism involving the capital allocation. Finally, the institution allocates capital to each of the divisions and they decide upon their optimal levels of risk. This problem is documented in Fig. 1.

In the next section we solve for the optimal capital allocation function in the situation where there is a single division manager with private information and where the risk taken is delegated at the divisional level.

3. Capital allocation with a single risky division

We first consider a setting in which the institution is composed of two divisions, where only one is strategic in the sense of having a manager with informational expertise. An example of this is where there is a specialized institution with lending activities but no trading book. The lending activity is strategic, but the other division is a treasury department with access only to a single riskless activity. In this case, we focus entirely on the single strategic division, and assume that any capital not invested in risky assets is held in the form of short term riskless securities earning the rate r_D.

The division manager is privately informed about his information parameter, θ.[12] Information is modeled as a single-dimensional draw, $\theta \in [\underline{\theta}, \bar{\theta}]$ given distribution function $F(\theta)$.

The problem is specified as a standard Bayesian game involving mechanism design (Fudenberg and Tirole, 1991, §7.2). This formulation assumes that the problem can be specified using a finite dimensional decision vector which in our context is the capital raised, C, as well as the risk chosen, σ. In addition the transfer is the capital allocation function, T. In this case the optimal mechanism may be derived by using the revelation principle (Myerson, 1979). The direct rev-

[11] At any chosen risk level, σ_i, Eq. (7) embodies the assumption that this same risk could be applied in an alternative firm to generate a larger share of cash flow revenues for the manager.

[12] In this section we drop the subscript i since we are only considering a single risky division.

elation mechanism is described by C, $\sigma(\hat{\theta})$ and $T(\hat{\theta})$, where $\hat{\theta}$ represents the division's *report* of information or "type." Equity capital is raised ex ante and hence is independent of the agents investment opportunities. The objective function of the agent (6) is separable in the decision and transfer variables as required.

The Bayesian Nash equilibria of the asymmetric information game are only those that can be supported by the *incentive compatibility* condition that $\hat{\theta} = \theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. We shall also be interested in the indirect mechanism where the capital allocation can be made a function of the decision variable. This requires our mechanism to be *implementable*. A necessary condition for this is if the Spence–Mirrlees 'sorting condition' holds

$$\frac{\partial}{\partial\theta}\left(\frac{\partial U}{\partial\sigma}\right) = \gamma\mu_\sigma > 0 \tag{8}$$

from Eq. (6) which holds by assumption.

The overall problem may therefore be expressed as maximizing expected EVA minus the compensation given to the manager or

$$\max_{T,\sigma(\hat{\theta}),C} I = E\big[\mu\big(\sigma\big(\hat{\theta}\big)\big)\theta - r_D\big(A\sigma\big(\hat{\theta}\big) - C\big) - r_E C - U\big], \tag{9}$$

subject to the incentive compatibility condition that the manager truthfully reports his private information in the optimal mechanism

$$\theta \in \arg\max_{\hat{\theta}} U\big(\sigma, \theta, \hat{\theta}\big) = \gamma\mu\big(\sigma\big(\hat{\theta}\big)\big)\theta - T\big(\hat{\theta}\big), \tag{10}$$

subject to reservation utility

$$U(\theta) \geqslant \underline{U} + \eta\mu\big(\sigma\big(\hat{\theta}\big)\big)\theta \tag{11}$$

and the regulatory constraint on overall firm equity capital

$$\alpha\sigma(\theta) \leqslant C. \tag{12}$$

In addition, we assume that the optimal solution satisfies $\sigma(\hat{\theta}) \geqslant 0$.

3.1. No outside options

We now solve problem (9) in the situation where the manager has outside options independent of the risk level chosen, i.e., $\eta = 0$. The standard approach in solving problem (9) is to first-convert the global incentive-compatibility condition (10) into a local representation. A necessary and sufficient condition for (10) to hold is that

$$U(\theta) = \underline{U} + \int_{\underline{\theta}}^{\theta} \gamma\mu\big(\sigma\big(\hat{\theta}\big)\big)\,\mathrm{d}\hat{\theta}, \tag{13}$$

and $\sigma(\theta)$ non-decreasing.[13] This also implies that the reservation utility constraint is binding only at the lower endpoint, $U(\underline{\theta}) \geqslant \underline{U}$. Using this representation, the optimal risk of the institution

[13] The sorting condition (8) along with monotonicity of the decision variable, σ, is the critical ingredient that is necessary to obtain this representation of the incentive compatibility conditions.

solves the following problem:

$$\max_{C,\sigma(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\mu\big(\sigma(\theta)\big)\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)\big(\alpha\sigma(\theta) - C\big) - U(\theta)\right] \mathrm{d}F(\theta), \quad (14)$$

subject to (13) where $\lambda(\theta)$ represents the Lagrange multiplier on the total capital constraint, (12).

With the exception of the capital constraint (12) the solution to this problem is standard within the mechanism design literature and is relegated to Appendix A. Proposition 1 gives the solution.

Proposition 1. *There exists a threshold value, $\theta^* \in [\underline{\theta}, \bar{\theta}]$ such that optimal risk for $\theta \in [\underline{\theta}, \theta^*]$ satisfies*

$$\mu_\sigma\big(\sigma(\theta)\big)\theta - r_D A = \frac{\gamma(1 - F(\theta))}{F'(\theta)}\mu_\sigma\big(\sigma(\theta)\big). \quad (15)$$

The optimal risk level for $\theta \in [\theta^*, \bar{\theta}]$ is constant, $\sigma(\theta) = \sigma^*$, where θ^* satisfies

$$\mu_\sigma(\sigma^*) \int_{\theta^*}^{\bar{\theta}} \theta \,\mathrm{d}F(\theta) = (r_E - r_D)\alpha + r_D A\big(1 - F(\theta^*)\big) + \gamma\mu_\sigma(\sigma^*)\left[\int_{\theta^*}^{\bar{\theta}} \theta \,\mathrm{d}F(\theta) - \big(1 - F(\theta^*)\big)\theta^*\right]. \quad (16)$$

To see how Proposition 1 results, it helps to compare this against the first-best case where θ is known and capital could be raised instantaneously. Here $C^* = \alpha\sigma^*$ and the first-order condition becomes

$$\mu_\sigma(\sigma^*)\theta - r_D A = (r_E - r_D)\alpha.$$

In this case, the optimal risk taking is determined both by the costs of debt as well as equity. However when equity capital cannot be raised continuously, then in the unconstrained region, the relevant opportunity cost is only that due to debt. This leads to more investment than with continuously adjustable equity capital. However, when information is also privately observed, the optimal second-best risk taking is reduced below the value which would be obtained using only the cost of debt capital. This can be seen from (15) by noting that the right-hand side is positive. Therefore, the marginal benefit of taking on increased risk at the second-best optimum is greater than the marginal cost. The extent of this deviation is greater for lower types. This is intentional on the part of the institution as it strives to make it more costly for the better divisions to misreport. The reason why the division has to be precluded from misreporting its type downward, has to do with the fact that given the nature of the constant reservation utility (11), the institution attempts to use the capital charge to extract the surplus of the division manager. Since higher θ divisions generate more surplus, an thus face larger capital charges, the manager of a higher type would prefer to misrepresent himself as a lower type. The only way to preclude this is to reduce the amount of investment directed for lower θ so that the manager does not misreport.

Equation (16) determines the optimal amount of capital that is raised. The left-hand side represents the benefit from raising α units of capital. This benefit is generated from taking on one additional unit of risk (standard deviation) in the states between θ^* and $\bar{\theta}$. This increases the cash flow in these states by μ_σ.

The right-hand side of Eq. (16) represents the costs of raising one additional unit of capital. First, the net cost of equity is $r_E - r_D$. This is so since equity capital can be thought of as being invested in the riskless asset. Second, the additional investment of A in the risky asset must be financed via deposits in the states between θ^* and $\bar{\theta}$. Third, the additional investment in the risky asset leads to additional payments to the divisional manager. This is captured in the third expression.

Thus, if the amount of equity capital raised is chosen so that Eq. (16) holds, then the marginal benefit of equity capital just equals its marginal costs.

Proposition 1 implies the following corollary.

Corollary 1.

$$C^* < \alpha\sigma^*(\bar{\theta}), \tag{17}$$

i.e. the risk limit is binding in some states.

Proof. Suppose that $C^* = \alpha\sigma^*(\bar{\theta})$. In this case $\theta^* = \bar{\theta}$. This implies that Eq. (16) cannot hold since the left-hand side is zero and the right-hand side is $(r_E - r_D)\alpha$. $\square$

3.2. Outside options

Now we consider the situation where the manager has outside options that increase with the information at a sufficiently high rate. Since most of the analysis mirrors that of the previous case, we shall be brief in deriving the results and then turn to an explanation of the differences and the implications.

In dealing with this case we follow the approach of Maggi and Rodriguez-Clare (1995), who show that one can define the manager's rent,

$$V(\theta) = U(\theta) - \eta\mu(\sigma)\theta. \tag{18}$$

Using this definition, note that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{\partial U}{\partial\theta} - \eta\mu_\sigma\theta\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} - \eta\mu = \gamma\mu - \eta\mu - \eta\mu_\sigma\theta\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}.$$

Since $\mathrm{d}\sigma/\mathrm{d}\theta$ is non-decreasing from the incentive compatibility constraint, it follows that a sufficient condition for $\mathrm{d}V/\mathrm{d}\theta < 0$ is that $\eta > \gamma$. The implication of this result is that the reservation utility constraint, $V(\theta) \geqslant \underline{U}$, is binding at the upper endpoint, $\theta = \bar{\theta}$.[14]

The equivalent problem to be solved is now

$$\max_{C,\sigma(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big[\mu\big(\sigma(\theta)\big)\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)\big(\alpha\sigma(\theta) - C\big) - \big(V(\theta) + \eta\mu\big(\sigma(\theta)\big)\theta\big)\Big]\,\mathrm{d}F(\theta), \tag{19}$$

subject to (13) and $V(\bar{\theta}) = \underline{U}$. The solution to this problem is given in the following proposition.

[14] Maggi and Rodriguez-Clare (1995) consider results intermediate to the two polar cases considered here and illustrate how countervailing incentives leads to situations of pooling amongst the types.

Proposition 2. *There exists a threshold value, $\theta^* \in [\underline{\theta}, \bar{\theta}]$ such that optimal risk for $\theta \in [\underline{\theta}, \theta^*]$ satisfies*

$$\mu_\sigma\big(\sigma(\theta)\big)\theta - r_D A = -\frac{\gamma F(\theta)}{F'(\theta)}\mu_\sigma\big(\sigma(\theta)\big). \tag{20}$$

The optimal risk level for $\theta \in [\theta^*, \bar{\theta}]$ is constant, $\sigma(\theta) = \sigma^*$, where θ^* satisfies

$$\mu_\sigma(\sigma^*)\int_{\theta^*}^{\bar{\theta}} \theta \,\mathrm{d}F(\theta) = (r_E - r_D)\alpha + r_D A\big(1 - F(\theta^*)\big)$$
$$+ \mu_\sigma(\sigma^*)\left[\gamma\big(\bar{\theta} - F(\theta^*)\theta^*\big) - \gamma\int_{\theta^*}^{\bar{\theta}} \theta \,\mathrm{d}F(\theta) - \eta\bar{\theta}\right]. \tag{21}$$

Proof. The proof is very similar to Proposition 1. The main differences are outlined in Appendix A. □

In comparison to the case without outside options, we see that once again there is a threshold level, θ^*, which is in general different with the capital constraint binding above this region. Below this region, there is a greater amount of risk taken in comparison with the case without outside options. The reason for this again has to do with information rent. In order to satisfy the form of the reservation utility constraint with increasing outside options, the institution must decrease the capital charge as θ increases, since the outside share of cash flow revenues is greater than the inside share. If this is the case, then managers with less desirable investment opportunities would proclaim more positive information in an attempt to capture this surplus, i.e., to ensure a lower capital charge. The only way to preclude this is for the institution to induce even greater degrees of distortion for higher θ which implies even more risk taking than in the first-best case. This is how the institution deals with the situation when managers outside options are increasing.

It is also instructive to contrast the conditions governing the threshold level and therefore the optimal amount of capital raised ex ante with those when the divisional manager has no increasing outside options. Comparing expression (16) to (21) we see that the only difference is in the form of the information rent term. In the following proposition, we show how this difference in information rent influences the optimal capital raised in the two cases.

Proposition 3. *When the manager has outside options, the range of states for which the risk limit is binding is greater than when outside options are non-existent.*

Proof. See Appendix A. □

The justification for the result in Proposition 3 derives from the commitment properties of capital. Compared to a symmetric information situation, underinvestment occurs in the case without outside options, while overinvestment occurs in the case with outside options. This means that in the latter case, by precommitting through the risk limit the institution mitigates the distortion problem that occurs in high states. In the former case, the risk limit plays no role in mitigating the distortion problem. This is the essential reason why the extent of limiting risk through the ex ante capital constraint is greater in the situation with outside options. In Section 5 we provide a numerical simulation for a specific example. This allows us to also derive some interesting comparative statics properties.

Our model also allows us to make predictions about capital utilization. First, for both the case of outside options and no outside options we get regions in which capital is not fully utilized. This accords well with actual experience in which risk limits are often underutilized. Second comparing the two cases we find that the region in which capital is not fully utilized is smaller in the case with outside options.

When private information does not generate outside opportunities, one might refer to managers with favorable information as "sandbagging." They are pretending to have less productive opportunities in order to earn excess rent. On the other hand, when managers outside opportunities are increasing, they can be thought to be "grandstanding," i.e., trying to signal as though their information is more favorable than it actually is. Preventing excess risk-taking is more important to the institution in the latter case than it is in the former, and this is why risk limits are stricter. The empirical prediction is thus that underutilization of risk limits occurs more frequently in situations in which managerial skills are more easily transferrable.

4. Implementation via EVA and RAROC

We now interpret our previous results by showing that the optimal risk level can be implemented by providing the manager with an appropriate incentive schedule and delegating the decision to him subject to the risk limit. We show that the incentive schedule can be interpreted as a *divisional EVA* compensation system, where economic capital is appropriately computed.

In this section we apply a version of the "taxation principle" first discussed by Rochet (1985) and Guesnerie (1995). This idea states that the direct revelation mechanism can be implemented by an indirect "tax" on the decision variable. The ability to do this is guaranteed when the Spence–Mirrlees sorting condition is satisfied as in our model.

First, note that since risk is bounded by σ^* in the optimal mechanism, the institution must impose a risk limit $\sigma \leqslant \sigma^*$ for all information types θ. Whenever the risk limit is not binding, consider the implementation of the optimal $\sigma(\theta)$ via the following (linear) incentive schedule:

$$\hat{T}(\hat{\theta}, \sigma) = \nu(\hat{\theta}) + \kappa(\hat{\theta})\sigma. \tag{22}$$

Suppose now that the division has "reported" $\hat{\theta}$, thereby determining the functional form of (22). Consider the sub-problem where the institution now allows the division to select the risk level by maximizing utility

$$\max_{\sigma} \gamma\mu(\sigma)\theta - \hat{T}(\hat{\theta}, \sigma). \tag{23}$$

Definition 1. The indirect mechanism $\hat{T}(\hat{\theta}, \sigma)$ *implements* the direct mechanism $\langle\sigma(\hat{\theta}), T(\hat{\theta})\rangle$ whenever the solution to (23), $\hat{\sigma}(\hat{\theta}) = \sigma(\hat{\theta})$ and $\hat{T}(\hat{\theta}, \hat{\sigma}(\hat{\theta})) = T(\hat{\theta})$ for all $\hat{\theta}$.

Using this definition, we now see that a necessary condition for implementation of the optimal second-best mechanism in the case with no outside options is that

$$\gamma\mu_\sigma\theta - \hat{T}_\sigma = 0$$

coincides with the optimal decision according to (15). Substituting for the optimality condition of (15) and the definition of $\hat{T}$ from (22) we arrive at

$$\kappa(\theta) = \gamma r_D A + \gamma^2 \mu_\sigma\big(\sigma(\theta)\big)\frac{1 - F(\theta)}{F'(\theta)}. \tag{24}$$

This result leads to the next proposition.

Proposition 4. *When the strategic divisional manager has outside options that do not depend on private information, the optimal mechanism may be implemented via a risk limit accompanied by a capital allocation schedule such that*

$$\hat{T}(\theta, \sigma) = \nu(\theta) + \left[r_D A + \mu_\sigma \gamma \frac{1 - F(\theta)}{F'(\theta)}\right]\sigma, \tag{25}$$

where

$$\nu(\theta) = \gamma\mu\big(\sigma(\underline{\theta})\big)\underline{\theta} - \kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma\big(\hat{\theta}\big)\,\mathrm{d}\kappa\big(\hat{\theta}\big) - \underline{U}. \tag{26}$$

Proof. The proof of Proposition 4 is in Appendix A. □

Next we illustrate how this capital allocation schedule can now be interpreted in terms of Economic Value Added from the divisional manager's perspective. Using the functional form for (25) in the utility function (6) gives a utility interpretation as a share of overall EVA minus an adjustment function

$$\begin{aligned} U(\theta) &= \gamma\mu(\sigma)\theta - \nu - \kappa\sigma \\ &= \gamma\mu(\sigma)\theta - \nu - \gamma r_D A\sigma - \gamma^2\mu_\sigma\frac{1 - F(\theta)}{F'(\theta)}\sigma \\ &= \gamma \mathrm{EVA} - \nu + \gamma(r_E - r_D)C - \gamma^2\mu_\sigma\frac{1 - F(\theta)}{F'(\theta)}\sigma. \end{aligned}$$

In this sense the manager essentially receives a share of overall EVA minus two adjustments independent of risk, ν, and $\gamma(r_E - r_D)C$, and a deduction for risk undertaken, represented by the last asymmetric information term.

We now extend these results to provide a Risk Adjusted Return on Capital interpretation. In this vein, the standard way in which RAROC is applied is such that

$$\mathrm{EVA} = (\mathrm{RAROC})(\mathrm{EC}),$$

where EC represents economic capital. Then the realized RAROC is compared to zero and shareholder value creation is achieved if and only if RAROC > 0. Alternatively, one can use the notion of Return on Risk Adjusted Capital or RORAC and define a hurdle rate, r^*, such that shareholder value creation is equivalent to RORAC $> r^*$.

It is common to define economic capital as the VaR criterion and apply the hurdle rate, r^*, to this amount as a charge for the usage of capital; therefore we set $\mathrm{EC} = \alpha\sigma$, giving a definition of RAROC as follows:

$$\mathrm{RAROC} = \frac{\mu\theta - r_D\sigma(A - \alpha) - \delta - r^*\mathrm{EC}}{\mathrm{EC}}, \tag{27}$$

where δ is an adjustment to the net income independent of risk, defined such that the numerator of the RAROC ratio equals EVA. Note in this formulation that the term $r_D\sigma(A-\alpha)$ is the fraction of the physical investment that is debt financed. Using the RAROC criterion, a division will continue to make risky investments as long as the RAROC of the marginal project is greater than zero. Proposition 5 now derives the hurdle rate applicable to the RAROC performance measure, for the case without outside options.

Proposition 5. *Suppose that RAROC is defined in terms of economic capital using a VaR criterion based on the amount of capital utilization. Then when the hurdle rate, r^*, is given by*

$$r^* = r_D\left[1+\frac{\gamma\mu_\sigma(1-F(\theta))}{r_D\alpha F'(\theta)}\right], \tag{28}$$

shareholder value is created whenever the change in the marginal RAROC for the incremental project is greater than zero and is optimized at the point where the marginal RAROC $= 0$.

Proof. Consider the RAROC of the marginal investment, given by the change in the numerator in Eq. (27) divided by the marginal change in economic capital, i.e., the denominator in (27)

$$\frac{\frac{\mathrm{d}\mu(\sigma)\theta}{\mathrm{d}\sigma} - r_D(A-\alpha) - \frac{\mathrm{d}(r^*\alpha\sigma)}{\mathrm{d}\sigma}}{\frac{\mathrm{d}(\alpha\sigma)}{\mathrm{d}\sigma}} = \frac{\mu_\sigma\theta - r_D(A-\alpha) - \alpha r^*}{\alpha}.$$

This expression is greater than zero whenever

$$\mu_\sigma\theta > r_D(A-\alpha) + \alpha r^*. \tag{29}$$

But from the optimal capital allocation mechanism,

$$\mu_\sigma\theta > \kappa/\gamma, \tag{30}$$

for all values of σ up to the optimal level. Therefore the optimum will be achieved using a RAROC hurdle rate as long the right-hand sides of Eqs. (29) and (30) are identical, or $r^*\alpha = \kappa/\gamma - r_D(A-\alpha)$, which is the same as (28). □

According to Eq. (28) the hurdle rate to be used for an additional unit of capital allocated to a manager is determined by two components. The first component is just the cost of debt, r_D. Note that for one additional unit of capital allocated, the division must clearly do better with the equity investment than the alternative which in this case is the return on riskless investments, r_D. The second component is $\gamma\mu_\sigma(1-F(\theta))/\alpha(r_D F'(\theta))$. This represents the increase in the required manager compensation due to an additional capital unit allocated.

It is interesting to note that the hurdle rate does not depend on the cost of equity, r_E. This is so since, at the time of the investment decision, the amount of equity is fixed. The correct hurdle rate for an additional unit of capital is therefore determined by the cost of funding the investment through deposits plus the additional management compensation.

The method for implementing EVA and RAROC in the case with outside options mirrors the previous case almost exactly. Once again we can define a capital allocation function,

$$\hat{T}(\hat{\theta},\sigma) = \nu(\hat{\theta}) + \kappa(\hat{\theta})\sigma.$$

Analogous to Proposition 4 the optimal capital allocation is then determined.

Proposition 6. *Under asymmetric information, the optimal mechanism with increasing outside options may be implemented via a risk limit accompanied by a capital allocation schedule such that*

$$\hat{T}(\theta,\sigma) = \nu(\theta) + \left[r_D A - \mu_\sigma \gamma \frac{F(\theta)}{F'(\theta)}\right]\sigma, \tag{31}$$

where

$$\nu(\theta) = \gamma\mu\big(\sigma(\underline{\theta})\big)\underline{\theta} - \kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma\big(\hat{\theta}\big)\,\mathrm{d}\kappa\big(\hat{\theta}\big) - \underline{U}. \tag{32}$$

It is therefore clear that RAROC is also implemented using economic capital, $\mathrm{EC} = \alpha\sigma$, with a hurdle rate defined as

$$r^* = r_D\left[1 - \frac{\gamma\mu_\sigma F(\theta)}{r_D \alpha F'(\theta)}\right]. \tag{33}$$

Once again, the hurdle rate depends on the cost of debt, adjusted for information rent. However, in this case the hurdle rate is lower than the case under symmetric information. Further, Proposition 5 applies to this case with the substitution of (33) as a hurdle rate. Note that the hurdle rate is below the cost of debt even though the firm can always invest at r_D. The reason for this is that the low hurdle rate forces the manager who would misrepresent his information to take excessive risks and the low hurdle rate increases the cost of misrepresentation.

It is instructive to consider the effect of increasing the required amount of equity capital, α, on the optimal hurdle rates. In both cases the effect is to move the hurdle rates closer to the cost of debt. This means that increased regulatory capital requirements can have very different effects depending on whether the divisional manager is sandbagging or grandstanding. In the former case, the hurdle rate decreases, with increased equity requirements, while in the latter case the hurdle rate increases. The hurdle rate deviates from the cost of debt only to satisfy the incentive compatibility constraints. As α increases the impact of a given choice of hurdle rate on the capital charge is larger and therefore the required return on equity can deviate less from the cost of debt.

5. A numerical example and comparative statics

We now illustrate the results of the previous section via a specific numerical example. This example is then applied to derive some comparative statics on the key economic parameters.

The general functional specification used in the simulation is

$$\mu(\sigma) = k\log(\sigma + 1). \tag{34}$$

For this specification, the optimal (interior) solution is linear in θ when this variable is uniformly distributed, then from Eq. (15) in Proposition 1,

$$\sigma^*(\theta) = \frac{k\theta(1+\gamma) - \gamma k\bar{\theta}}{r_D A} - 1,$$

in the case where the manager has no outside options. When the conditions are satisfied for dominant outside options, then,

$$\sigma^*(\theta) = \frac{k\theta(1+\gamma) - \gamma k\underline{\theta}}{r_D A} - 1,$$

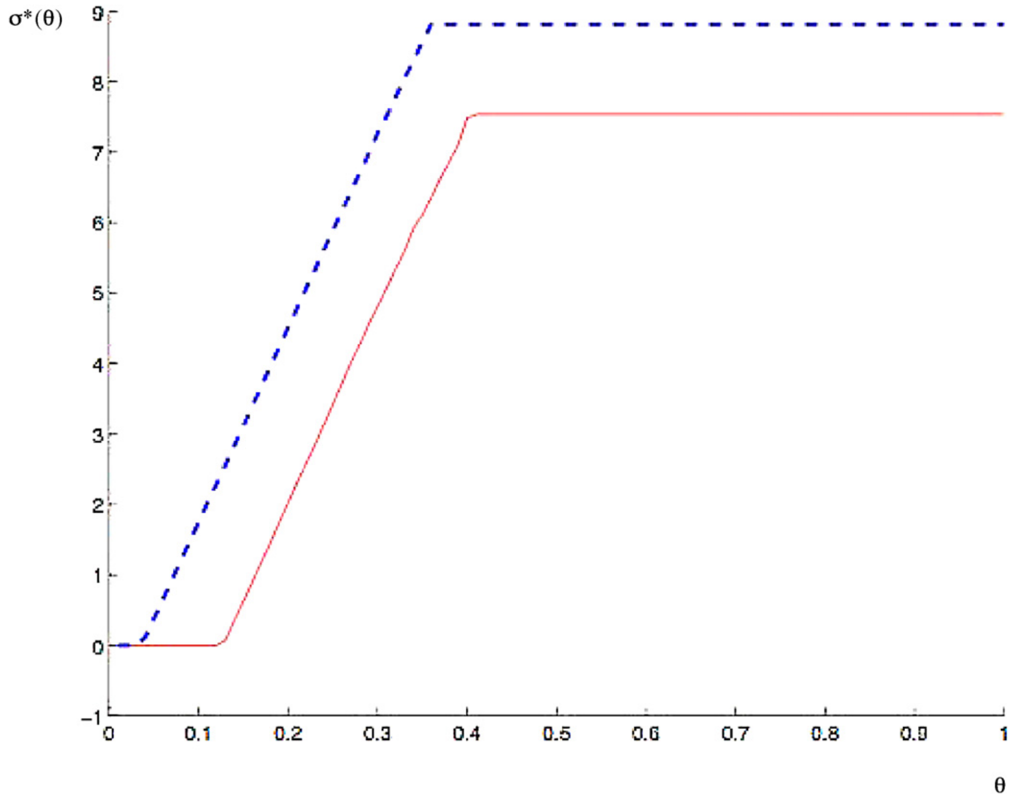


Fig. 2. Optimal mechanisms.

Table 1
Parameter values used in the numerical example

Parameter	Value
Cost of debt	$r_D = 0.04$
Cost of equity	$r_E = 0.05$
Confidence level	$\alpha = 2.33$
Reservation utility	$\underline{U} = 0.4$
Financing requirement	$A = 1$
Information distribution	$\theta \sim \text{unif}[0, 1]$
Equity ownership fraction	$\gamma = 0.1$
Productivity parameter	$k = 1$
Outside options	$\eta = 0$ or $\eta = 0.2$

also linear in θ. Table 1 summarizes the other choices for parameter values.

Even though the optimal solution for risk levels can be solved for analytically, it is still necessary to perform a numerical optimization to identify the optimal risk limits. Figure 2 provides the optimal solution in the cases with and without outside options. In this graph, the vertical axis shows $\sigma^*(\theta)$, with the optimal caps on risk taking activity. The red (solid) line shows the case with no outside options, while the blue (dashed) line indicates that with the outside options. This

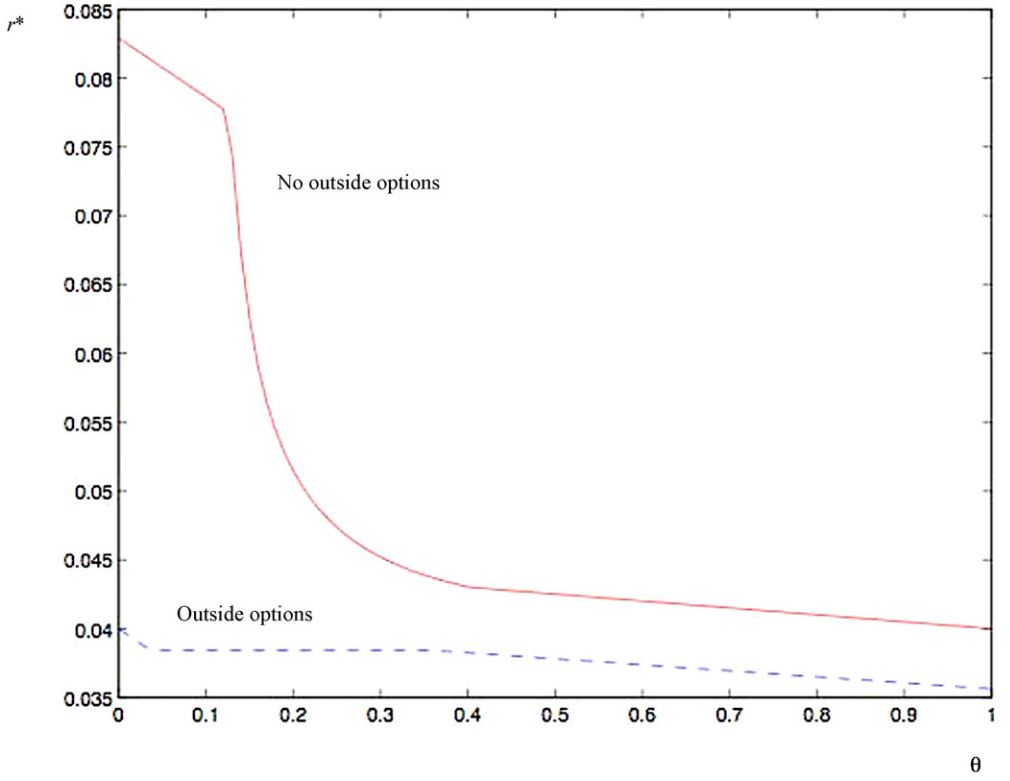


Fig. 3. Hurdle rates.

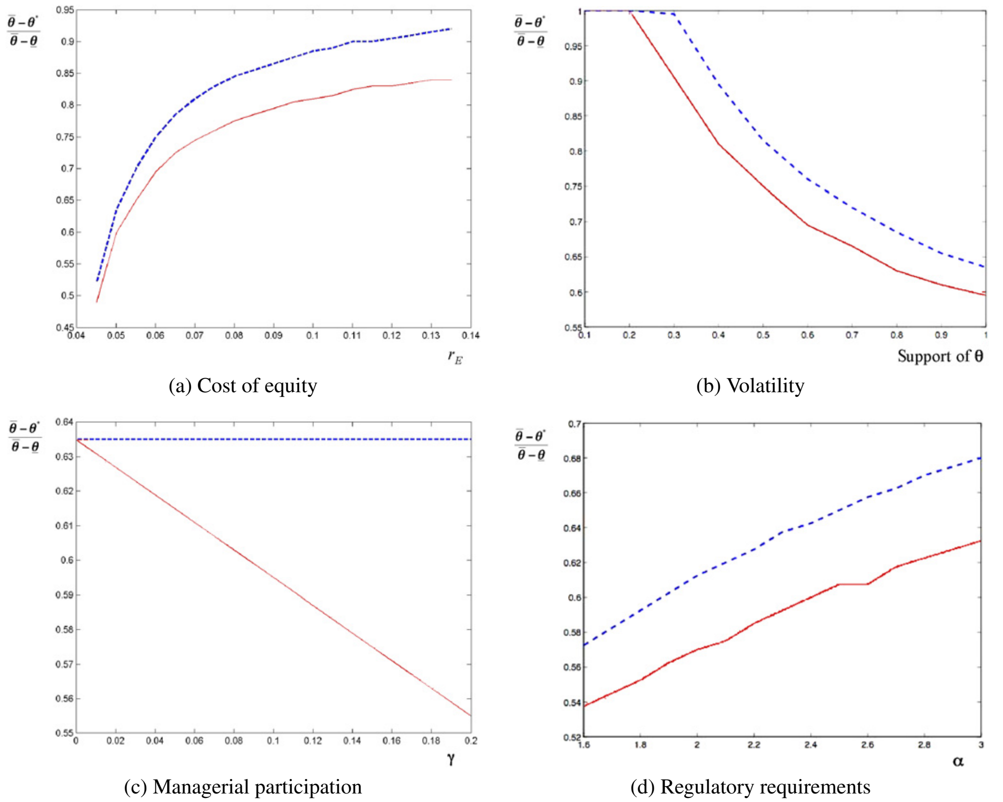
figure confirms the results of Proposition 3, showing that even though capital raised is greater in the case with outside options, the number of states where the risk limits are binding is greater.

Figure 3 illustrates that the respective hurdle rates are decreasing in θ for both cases with and without outside options. In the case without outside options, the hurdle rate is above the cost of debt, $r_D = 0.04$, and decreases significantly with θ. The reason for this is related to incentive compatibility. Recall that with no outside options, managers desire "sandbagging" by misrepresenting their information downwards. To counter these desires, the hurdle rates are high, or the manager is charged with a high cost of capital for low θ. As expected for high information parameters, the hurdle rate approaches the cost of debt.

For the case with outside options, managers desire "grandstanding" by misrepresenting their information in an upwards direction. Now, hurdle rates are biased downwards, so that they are induced to overinvest in risky assets, which is more costly for managers observing lower values of θ. Therefore our results show that optimal hurdle rates are downward sloping functions of investment opportunities.

5.1. Comparative statics

Our model helps to shed light on the effect of risk-based regulation on banks' abilities to take advantage of investment opportunities. Since capital is raised ex ante, such risk-based regulation implies that the bank is unable to realize all positive NPV projects. In the following we calcu-



(a) Cost of equity (b) Volatility

(c) Managerial participation (d) Regulatory requirements

Fig. 4. Percent constrained comparative statics. These graphs indicate the impact of the respective parameters on the degree to which investment activity is constrained as measured by $(\bar\theta-\theta^*)/(\bar\theta-\underline{\theta})$.

late the percentage of states in which the bank is constrained by the limited amount of capital. The main parameters driving this measure of investment constraints are the equity premium, the volatility of investment opportunities, the managerial incentive contract and the regulatory equity requirement. Figure 4 presents the effects of these parameters.

Cost of Equity: As the cost of equity increases, not surprisingly the amount of equity raised decreases for both situations, holding constant the bank's investment opportunities (see Fig. 4a). This effect is greater for lower values of the cost of equity. Correspondingly the set of states in which the capital constraint binds increases with the cost of equity, in a concave fashion as depicted in Fig. 4. This is intuitive because when the cost of equity is near the cost of debt the impact of having "excess capital" which is invested in the riskless asset is less costly ex ante.

Information Uncertainty: We also considered the effect of changes in the support of the distribution on θ (see Fig. 4b). By considering a series of "mean preserving spreads" in the distribution, we are altering the magnitude of real investment options of the bank. In this case, when the distribution is very "tight" then less capital is raised, since there are fewer states where risk taking is significantly profitable on the upside. Moreover, the number of states where the capital constraint is binding also increases as the distribution becomes more concentrated. In fact, with a very tight distribution of the information parameter, θ, there is little to be gained from varying risk-taking activities and capital binds in all states.

Managerial Share: As might be expected increasing managerial participation in the two cases works differently (see Fig. 4c). Without outside options, managerial sharing increases the extent of the underinvestment problem and the capital constraint is less important. In the case with increasing outside options, managerial sharing reduces the rent as well as increasing the extent of capital raised so that the two effects offset one another.

Required Equity Capital: The impact of making equity capital regulatory requirements more stringent, i.e., increasing α, unambiguously causes the amount of capital raised initially to increase (see Fig. 4d). This is necessary in order to support the optimal level of risk-taking. In addition the impact in terms of the percent of states affected by the limits is also increasing with α.

6. Capital allocation in a firm with multiple risky divisions

We now consider the problem of a firm with multiple risky divisions under incomplete information. As before we derive the optimal mechanism and show how it can be implemented in a delegation environment. The basic aspects of the previous model are preserved in this environment, however they need to be interpreted carefully. The implications for large diversified institutions are also considered.

Suppose that there are n divisions in the firm. Cash flows are as given in Eq. (1). Define the overall risk of the portfolio of investments from all divisions to be σ_p where

$$\sigma_p(\sigma_1, \sigma_2, \ldots, \sigma_n)^2 = \sum_i^n \sum_j^n \sigma_i \sigma_j. \tag{35}$$

For notational convenience, where there is no confusion, we denote the n-dimensional vector of decisions and types as $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ and $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$. The vector with a specific value substituted for element i is denoted by $(\hat{\theta}_i, \theta_{-i}) = (\theta_1, \theta_2, \ldots, \theta_{i-1}, \hat{\theta}_i, \theta_{i+1}, \ldots, \theta_n)$ For simplicity, we assume that information θ_i pertains only to division i and that it is independent across divisions.

The problem is formulated as a direct revelation game in which divisions each report the value of their private information subject to a Bayesian Nash incentive compatibility condition.[15] The direct revelation mechanism with multiple divisions is defined by the functions $\sigma_i(\theta)$, $i = 1, \ldots, n$, denoting the risk level for division i as a function of the joint information of all divisions in the firm, and $T_i(\theta)$, $i = 1, \ldots, n$, the capital allocation function. Information is represented by the joint distribution function, $F(\theta) \in [\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2] \times \cdots \times [\underline{\theta}_n, \bar{\theta}_n]$, exhibiting independence with respect to (θ). Given the Bayesian Nash structure of the problem, we use the following notation: a bar over a variable (e.g., $\bar{\mu}_i$) indicates that expectations are taken by division i with respect to its own information set, which consists of knowledge of θ_i and none of the other divisional θ_j parameters. With these features, the multidivisional problem under joint

[15] Darrough and Stoughton (1989) discuss a joint venture mechanism design problem in which there is mutivariate private information. They derive the functional form of optimal screening mechanisms. Mookherjee and Reichelstein (1992) discuss the formulation of a multi-agent screening problem and the conditions under which implementation via *dominant* strategies is possible. Unfortunately due to the joint dependence through the portfolio effects their “condensation” condition does not hold. As a result we consider Nash implementation.

asymmetric information is

$$\max_{\sigma_i(\theta),T_i(\theta),C} I = E\left[\sum_i \mu_i\big(\sigma_i(\theta)\theta_i\big) - r_D\left(\sum_i A_i\sigma_i(\theta) - C\right) - r_E C - \sum_i \bar{U}_i(\theta_i)\right] \tag{36}$$

subject to

$$\theta_i \in \arg\max_{\hat{\theta}_i} \gamma_i\bar{\mu}_i\big(\sigma_i\big(\hat{\theta}_i,\theta_{-i}\big)\big)\theta_i - \bar{T}_i\big(\hat{\theta}_i,\theta_{-i}\big), \tag{37}$$

$$\bar{U}_i = \gamma_i\bar{\mu}_i\big(\sigma_i(\theta)\theta_i\big) - \bar{T}_i(\theta) \geqslant \underline{U}_i + \eta_i\bar{\mu}_i\big(\sigma_i(\theta)\big)\theta_i, \tag{38}$$

$$C \geqslant \alpha\sigma_p\big(\sigma(\theta)\big). \tag{39}$$

The objective function of the institution, (36) reflects the need to take expectations over the joint distribution of divisional types, $F(\theta)$. The set of equations embodied in (37) indicates that each division reports the "true" value of private information while taking the truthful report of the other divisions as given. Similarly in (38) each division must take expectations over the others information parameter. The total capital regulatory constraint, (39) is as before.

Given the structure of the problem, it is fairly straightforward to apply analysis similar to that of Section 3, except with respect to all divisions. The incentive compatible representation condition for Eqs. (37) is

$$\bar{U}_i(\theta_i) = \underline{U}_i + \int_{\underline{\theta}_i}^{\theta_i} \gamma_i\bar{\mu}\big(\sigma_i\big(\hat{\theta}_i,\theta_{-i}\big)\big)\,\mathrm{d}\hat{\theta}_i, \tag{40}$$

and the expectation (over θ_{-i}) of the designated risk level for division i, $\bar{\sigma}_i(\theta_i,\theta_{-i})$, is non-decreasing in θ_i.

Again, when $\eta_i = 0$ so that division i enjoys no increasing outside options, by analogy with Eq. (15), we therefore find that the optimal joint levels of risk obtained in the multidivisional mechanism satisfies

$$\mu_{i\sigma}(\sigma_i)\theta_i - r_D A_i - \gamma_i\mu_{i\sigma}\left[\frac{1-F_i(\theta_i)}{F_i'(\theta_i)}\right] - \lambda(\theta)\alpha\frac{\partial\sigma_p}{\partial\sigma_i} = 0, \tag{41}$$

where F_i denotes the marginal distribution of θ_i, and $\lambda(\theta_1,\theta_2)$ is the Lagrange multiplier on the total capital constraint, which satisfies[16]

$$\int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\alpha\,\mathrm{d}F(\theta) = (r_E - r_D)\alpha. \tag{42}$$

6.1. Incremental value at risk

To interpret these results, we now utilize the concept of *incremental value at risk* (IVaR) as defined here.

[16] See the proof of Proposition 1 in Appendix A.

Definition 2. The incremental value at risk, $\varsigma_i(\sigma)$ for division i is defined as

$$\varsigma_i(\sigma) = \alpha\sigma_i \frac{\partial\sigma_p}{\partial\sigma_i}. \tag{43}$$

The incremental value at risk can be interpreted in terms of the regression coefficient from a regression of the cash flows of division i on the institution's overall portfolio. Specifically, if β_{ip} is the regression coefficient, then $\varsigma_i = \alpha\beta_{ip}\sigma_p$. Not surprisingly therefore, the incremental value at risk has the property that $\alpha\sigma_p = \sum_i \varsigma_i$, i.e., the sum of the IVaRs equals the institution's overall VaR.

Substituting this definition into the first-order condition above gives the following representation for the optimal investment decision of each division:

$$\mu_{i\sigma}(\sigma_i)\theta_i - r_D A_i - \gamma_i \mu_{i\sigma}\left[\frac{1 - F_i(\theta_i)}{F_i'(\theta_i)}\right] - \lambda(\theta)\frac{\varsigma_i}{\sigma_i} = 0. \tag{44}$$

That is, investment occurs up to the point where the marginal increase in expected cash flows is balanced by the costs of capital, for both the physical investment required, represented by the second term in Eq. (44), as well as the incremental contribution to the overall risk of the institution, represented by the last term in Eq. (44). In addition, due to asymmetric information, the marginal benefit is reduced by the rent paid out to the manager of division i, as in the case of the single risky division. This is represented in the third term of Eq. (44).

6.2. Implementation

In order to operationalize the above mechanism through an indirect mechanism where the risk level is delegated to each divisional manager, we propose the following multivariate mechanism:

(1) the central authority asks each division manager to make a report of their information, $\hat{\theta}_i$;
(2) based on the joint set of reports, the central authority selects a capital allocation function, $\hat{T}_i(\hat{\theta}, \sigma_i)$ a function of the joint reports and the risk level for division i;
(3) delegates the decision, σ_i to each divisional manager i so as to solve their individual economic value added as in

$$\max_{\sigma_i} \mathrm{EVA}_i = \gamma_i \mu_i(\sigma_i)\theta_i - \hat{T}_i\big(\hat{\theta}, \sigma_i\big), \tag{45}$$

where

$$\hat{T}_i\big(\hat{\theta}, \sigma_i\big) = \bar{v}_i\big(\hat{\theta}_i\big) + \kappa_i\big(\hat{\theta}\big)\sigma_i. \tag{46}$$

That is, each division is presented with a linear capital allocation schedule with a fixed component, $\bar{v}_i$ that does not depend on risk taken, the reports or the action of the other divisions. The risk charge, κ_i, however is impacted by the joint set of reports. Generally, with better private information of other divisions, the risk charge for division i will be greater. This induces a kind of internal capital market within the financial institution.

Proposition 7 establishes the functional form of this optimal indirect mechanism. Its proof is essentially the same as that of Proposition 4.

Proposition 7. *The optimal multidivisional capital allocation mechanism can be implemented by a modified IVaR schedule such that*

$$\kappa_i(\theta) = \gamma_i r_D A_i + \gamma_i^2 \mu_{i\sigma}\big(\sigma_i(\theta)\big)\frac{1 - F_i(\theta_i)}{F_i'(\theta_i)} + \gamma_i \lambda(\theta)\frac{\varsigma_i(\theta)}{\sigma_i(\theta)} \tag{47}$$

and

$$\begin{aligned}\bar{\nu}_i(\theta_i) = {} & \gamma_i \bar{\mu}_i\big(\sigma_i(\underline{\theta}_i, \theta_{-i})\big)\underline{\theta}_i - \bar{\kappa}_i(\underline{\theta}_i, \theta_{-i})\bar{\sigma}_i(\underline{\theta}_i, \theta_{-i}) \\ & - \int_{\underline{\theta}_i}^{\theta_i} \bar{\sigma}_i\big(\hat{\theta}_i, \theta_{-i}\big)\,\mathrm{d}\bar{\kappa}_i\big(\hat{\theta}_i, \theta_{-i}\big) - \underline{U}_i,\end{aligned} \tag{48}$$

where $\bar{\kappa}_i$ denotes expectations of κ_i with respect to θ_{-i} and likewise $\bar{\sigma}_i$ also denotes expectations with respect to the information variables of division other than i.

The only major difference between the single risky and multiple risky division capital allocation schedules lies in the last term in Eq. (47). This is the IVaR term and indicates where the interactive effect is present. Recall that in the single risky division case, the capital allocation mechanism is utilized for risk selection only when the capital constraint is not binding. Here in the multiple risky division problem the capital allocation mechanism will need to be utilized even when the capital constraint is binding. This is because even if capital is constrained, it must be allocated optimally across divisions and the externality of one division's risk choice on the other must be internalized. The 'price' of risk, $\lambda\varsigma_i/\sigma_i$ has both a common and a divisional specific component. The common component which will impact both divisions evaluation is λ, the shadow price of the capital constraint.[17] If one division contributes more in terms of IVaR than another division, its own internal price for risk will be adjusted higher. However each division takes its own risk charge as fixed and applies it to its own risk level to make the optimal joint decision from the point of the institution.

6.3. RAROC

The goal of a RAROC hurdle rate in the multiple risky division case is to get each division acting in its own interest to select the overall optimal level of aggregated risk for the institution. As in the single division case, this requires an initial phase in which the hurdle rate is established based on reports or selections of capital allocation mechanisms by all divisions. However, the divisions should be judged using only their own contribution to risk. Therefore we utilize the IVaR concept in our definition of RAROC in the multidivisional situation. Let $\varsigma_i^* = \varsigma_i(\theta)$ and $\sigma_i^* = \sigma_i(\theta)$ stand for the values of the IVaR and risk levels at the *optimal* risk decisions based on truthful reports. Then define RAROC as

$$\mathrm{RAROC} = \frac{\mu_i(\sigma)\theta_i - r_D\sigma_i(\varsigma_i^*/\sigma_i^*)(A_i\sigma_i^*/\varsigma_i^* - 1) - \bar{\delta}_i - r^*[\varsigma_i^*/\sigma_i^*]\sigma_i}{[\varsigma_i^*/\sigma_i^*]\sigma_i}. \tag{49}$$

Considering the RAROC of the marginal project, there are two cases. Under typical circumstances, the IVaR will be positive, indicating that cash flows of division i are positively correlated with overall cash flows. Then risky investments will continue to be made as long as

[17] If the capital constraint is not binding, for low joint values of θ_i and θ_j, the price will be zero.

$\mu_{i\sigma}\theta_i - r_D(A_i - \varsigma_i^*/\sigma_i^*) - r^*(\varsigma_i^*/\sigma_i^*) > 0$. This is consistent with optimality, (41), if the hurdle rate, r^* is given by $r^*(\varsigma_i^*/\sigma_i^*) + r_D(A_i - \varsigma_i^*/\sigma_i^*) = \kappa_i/\gamma_i$. Therefore we find from (47) that the RAROC hurdle rate satisfies

$$r^* = r_D + \gamma_i \mu_{i\sigma} \frac{1 - F(\theta_i)}{F'(\theta_i)} (\sigma_i^*/\varsigma_i^*) + \lambda(\theta). \tag{50}$$

On the other hand, if the IVaR is negative, indicating that division i is serving as a type of "hedge" for the aggregate risk of the institution, then it turns out that the optimal investment policy is to invest whenever the marginal change in the numerator of (49) is negative: $\mu_{i\sigma}\theta_i - r_D(A_i\sigma_i^*/\varsigma_i^* - 1) - r^*(\varsigma_i^*/\sigma_i^*) < 0$.

As in the single risky division case, the hurdle rate is related to the cost of debt capital, since this is still the alternative non-strategic investment opportunity. There is also another positive increment to the hurdle rate from asymmetric information in this case. However an important difference is that there is now an interactive effect, which is a charge for the impact of the capital constraint, λ, when it is binding. This charge is identical for all divisions within the firm, and represents the cost of taking capital away from its alternative uses.

6.4. The effect of intra-firm diversification

An important difference between a financial institution with capital regulation and a standard firm is that diversification of activities can matter. That this can lead to lower levels of capital requirements at the institutional level has been pointed out in the literature (Perold, 2005). We focus instead on the implications that this has at the divisional level. As we have demonstrated the capital charge faced by division i from its risk decision is equal to σ_i which can be written as $\gamma_i\lambda(\varsigma_i^*/\sigma_i^*)\sigma_i$. We now focus separately on the terms ς_i^*/σ_i^* and λ as the number of risky divisions increases in a diversified financial institution.

We first discuss the coefficient ς_i^*/σ_i^*, which can be interpreted as the coefficient multiplying division is own risk to determine the economic capital allocated to it. To study the limit of this coefficient as the number of divisions gets large, we specify a factor structure for the stochastic component of cash flows in Eq. (1) as follows:

$$z_i = \beta\big(R_m - \bar{R}_m\big) + \epsilon_i, \tag{51}$$

which represents a common systematic factor loading, e.g., the market return where we have normalized these variables so that $E(R_m) = \bar{R}_m$, $E(\epsilon_i) = 0$, where the ϵ_i for each division is independent of other divisions and that of the systematic factor. We also assume that the variances of R_m and ϵ_i are such that

$$\beta^2\sigma_m^2 + \sigma_\epsilon^2 = 1, \tag{52}$$

so that the standard deviation of z_i satisfies our assumption that it equals one.

Let σ^* denote the optimal risk level for a bank with a single division given information level θ. Consider replicating this division within a multidivisional institution where each of the n division's optimal risk level is determined by the same θ and scaled by the factor k/n, i.e., each division optimally selects risk $\sigma_i = k\sigma^*/n$. In this case, the benefit of diversification takes place only through cash flows, not through the productivity parameters. The following proposition establishes the limiting value for the coefficient ς_i^*/σ_i^* under these conditions.

Proposition 8. *When divisional cash flows satisfy a common factor model given by (51), and all observe a common productivity, θ, the limit of the economic capital coefficient,*

$$\lim_{n\to\infty} \varsigma_i^*/\sigma_i^* = \alpha\beta\sigma_m < \alpha. \tag{53}$$

Proof. See Appendix A. □

Most importantly this proposition shows that in the limit with multiple divisions, considering an infinite replication, each division will face a lower economic capital coefficient than as a stand alone entity, where this coefficient is equal to α. This means that they are less sensitive to their own risk than as a stand alone entity. This proposition shows that the diversification benefits are related to the proportion of the cash flows generated by the systematic factor. When virtually all the cash flows are idiosyncratic, $\beta\sigma_m$ is near zero and the economic capital coefficient becomes very small.

Now we turn to the situation where benefits to diversification can be generated through diversity of productivity information across divisions. We illustrate the effect on the hurdle rate utilized in the RAROC evaluation. In this case, consider a situation where all n divisions are identical in terms of expected cash flow functions, $\mu(\sigma_i)\theta_i$, with identical distribution functions, $F(\theta_i)$. The only differences in risk levels taken is through the random draws of the productivity parameter, θ_i. Given that Eq. (44) holds for each division i, we can see that $\lambda(\theta)$, only depends on the empirical distribution of θ_i for the sample of n draws. This means that θ does not depend which division draws a certain productivity parameter; only the frequency with which that parameter is drawn.

Taking the limit as the empirical frequency distribution converges to the theoretical distribution, which holds due to assumption of i.i.d. productivity parameters across divisions, this means that in the limit as n goes to infinity, λ depends only on the functional form, F, of the theoretical probability distribution, which is a constant. From Eq. (42), λ is given by

$$\lambda = r_E - r_D. \tag{54}$$

This result is stated in the following proposition.

Proposition 9. *When divisions are identical with respect to their risky investment technologies and differ only in their productivity parameters, which are i.i.d. the RAROC hurdle rate in the case without outside options converges to*

$$r^* = r_E + \gamma_i \mu_{i\sigma} \frac{1 - F(\theta_i)}{F'(\theta_i)} (\sigma_i^*/\varsigma_i^*). \tag{55}$$

Proof. Substitute (54) into Eq. (50). □

Proposition 9 therefore establishes important differences between capital allocation in a single risky division case and the multidivisional case when capital cannot be raised instantaneously. The inflexibility of capital creates an inefficiency in a single risky division, due to deadweight costs when it is not profitable to utilize capital fully, and is constrained on the other hand when it is most profitable. In a large multidivisional firm, this discrepancy is mitigated and in the limit both of these inefficiencies disappear, leaving only that due to asymmetric information. As a result, the optimal hurdle rate used in capital budgeting is based on the cost of equity instead of the cost of debt.

6.5. Increasing outside options

Most of the analysis when the manager of division i has increasing outside options, $\eta_i > \gamma_i$ is handled analogously with the single division case. All of the results go through concerning the form of the capital allocation schedule; now the risk charge for allocation to division i becomes

$$\kappa_i(\theta) = \gamma_i r_D A_i - \gamma_i^2 \mu_{i\sigma}\big(\sigma_i(\theta)\big)\frac{F_i(\theta_i)}{F_i'(\theta_i)} + \gamma_i \lambda(\theta)\frac{\varsigma_i(\theta)}{\sigma_i(\theta)}. \tag{56}$$

The RAROC hurdle rate in this case should be defined as

$$r^* = r_D - \gamma_i \mu_{i\sigma}\frac{F(\theta_i)}{F'(\theta_i)}(\sigma_i^*/\varsigma_i^*) + \lambda(\theta). \tag{57}$$

As in the single risky division case, this is lower than the hurdle rate without outside options in order to promote higher risk taking.

7. Implications and conclusions

Optimal capital allocation and the identification of appropriate performance benchmarks in the presence of capital market frictions have significant consequences for financial institutions. In this environment, corporate executives and boards of directors must decide on real investment options, taking overall risk into account. While many advances have been made in measuring and analyzing statistical properties of prices and cash flows, few results exist on integrating these measures for decision rules. This paper presents a consistent framework for applying EVA and RAROC for shareholder value optimization in a multidivisional financial institution.

Important features of the model are asymmetric information at the divisional level, outside options of managers and coordination of risk-taking activities. In such a framework it is shown that RAROC and EVA can be justified in terms of a precise mechanism design. The important issues that we have focused on include how much equity capital to raise ex ante, the use of fixed risk limits, a linear capital charge schedule, the measurement of economic capital and the required hurdle rate to employ.

A simplifying assumption of the model is that the cost of equity is constant at the time when the institution determines the amount of equity capital to be raised. However, since firms must satisfy a VaR constraint, this assumption can be justified if investment decisions are always made such that this constraint is binding. If not, an iteration would be required where the cost of equity reflects the realized expected systematic risk undertaken by the institution. Nevertheless, in this more general setting the firm would still take the cost of equity and debt as given when running the optimal capital allocation mechanism.

It is shown that the linear incentive schedule charges each division manager with a cost of capital multiplied by the division's actual economic capital utilization as measured by the contribution to value at risk. Further, this capital allocation mechanism also implies an appropriate hurdle rate that the return on economic capital must overcome for an investment to be optimal. In a multidivisional setting, the central authority of the institution plays an important role in designing the appropriate channels of communication and setting the transfer price for internal capital. Nevertheless, the actual investment and risktaking decisions can be delegated in an independent manner to the respective divisions.

In the multidivisional context there are significant portfolio effects that can be achieved through diversification. We identify the impact on economic capital charge as well as on the hurdle rate. We find that economic capital is optimally set equal to incremental value at risk which is equal to the divisions own risk times a coefficient, which is less than one. This coefficient declines when the overall risks of the institution are less systematic. Another diversification effect takes place due to asymmetric information. When investment productivities are independent across divisions, the hurdle rate tends toward the common cost of equity of the institution, instead of reflecting divisional specific risks. Interestingly this supports the idea that hurdle rates in a diversified financial institution are more nearly equal than would be predicted in perfect markets settings.

An interesting extension of this paper would be to analyze the implications for organizational structure. Such issues would include the extent to which a bank holding companies should be used, subsidiary-specific financing arrangements and merger and divestiture implications. Further, we have assumed that capital can flow freely between business units. In practice capital mobility is restricted by legal and managerial constraints, especially if different business units are located in different jurisdictions. Thus, one needs to explore the effect of such restrictions on capital mobility on the amount of capital to be raised ex ante and on the capital allocation process. Also, extending this paper's one-period static setting to a dynamic model, in which capital can be reallocated over time and new equity can be raised externally would be an important step towards developing a conceptual framework for capital allocation decisions in practice.

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Appendix A

A.1. Proof of Proposition 1

The solution to this problem is standard within the screening literature (Guesnerie and Laffont, 1984).

Using the representation of the incentive compatibility constraints (13), and integration-by-parts, we find that

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\,\mathrm{d}F(\theta) = U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)\,\mathrm{d}U(\theta) = U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu\big(\sigma(\theta)\big)F(\theta)\,\mathrm{d}\theta$$

$$= U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu\big(\sigma(\theta)\big)\big(1 - F(\theta)\big)\,\mathrm{d}\theta.$$

We now rewrite (14) as follows:

$$\max_{C,\sigma(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big[\mu\big(\sigma(\theta)\big)\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)\big(\alpha\sigma(\theta) - C\big)$$

$$- \gamma\mu\big(\sigma(\theta)\big)\frac{1-F(\theta)}{F'(\theta)}\Big]\,\mathrm{d}F(\theta) - \underline{U}. \tag{A.1}$$

The determination of the optimal $\sigma(\theta)$ can now be accomplished in a pointwise manner. This gives the following set of first-order conditions:

$$\mu_\sigma(\theta) - r_D A - \lambda(\theta)\alpha - \gamma\mu_\sigma(\theta)\frac{1-F(\theta)}{F'(\theta)} = 0 \tag{A.2}$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} \big[r_D - r_E + \lambda(\theta)\big]\,\mathrm{d}F(\theta) = 0. \tag{A.3}$$

When the constraint on capital is non-binding, $\lambda = 0$ and Eq. (A.2) gives the first condition, (15), in the proposition. Now write (A.3) as

$$(r_E - r_D)\alpha = \int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\alpha\,\mathrm{d}F(\theta),$$

and notice that over the range where the capital constraint is binding, $\lambda(\theta) > 0$, $\sigma(\theta)$ and hence $\mu_\sigma(\sigma(\theta))$ is also constant. Because $\sigma(\theta)$ must be non-decreasing, the capital constraint is only binding over an interval $[\theta^*, \bar{\theta}]$. From (A.2),

$$\lambda(\theta)\alpha = \mu_\sigma\theta - r_D A - \gamma\mu_\sigma\frac{1-F(\theta)}{F'(\theta)},$$

in which case

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\alpha\,\mathrm{d}F(\theta) &= \int_{\theta^*}^{\bar{\theta}} \left\{\left[\theta - \gamma\frac{1-F(\theta)}{F'(\theta)}\right]\mu_\sigma - r_D A\right\}\mathrm{d}F(\theta) \\
&= (1-\gamma)\mu_\sigma \int_{\theta^*}^{\bar{\theta}} \theta\,\mathrm{d}F(\theta) + \gamma\mu_\sigma\big(1 - F(\theta^*)\big)\theta^* - r_D A\big(1 - F(\theta^*)\big) \\
&= (r_E - r_D)\alpha.
\end{aligned}$$

Equation (16) is a rearrangement of this latter expression. □

A.2. *Outline of proof of Proposition 2*

First, we use the definition of V to show that

$$\int_{\underline{\theta}}^{\bar{\theta}} V(\theta)\,\mathrm{d}F(\theta) = V\big(\bar{\theta}\big) - \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu\big(\sigma(\theta)\big)F(\theta)\,\mathrm{d}\theta + \bar{\theta}\eta\mu\big(\sigma\big(\bar{\theta}\big)\big) - \int_{\underline{\theta}}^{\bar{\theta}} \eta\theta\mu\big(\sigma(\theta)\big)\,\mathrm{d}F(\theta).$$

Then this is substituted into the objective function of (19) which can be rewritten as

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[\mu\big(\sigma(\theta)\big)\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)\big(\alpha\sigma(\theta) - C\big) + \gamma\mu\big(\sigma(\theta)\big)\frac{F(\theta)}{F'(\theta)}\right] \mathrm{d}F(\theta)$$
$$- V(\bar{\theta}) - \bar{\theta}\eta\mu\big(\sigma(\bar{\theta})\big).$$

Equation (20) is given by the pointwise derivative of this expression when $\lambda(\theta) = 0$. Equation (21) is derived in a manner similar to that of Proposition 1 from the derivative with respect to C

$$\int_{\underline{\theta}}^{\bar{\theta}} \big[-(r_E - r_D) + \lambda(\theta)\big]\,\mathrm{d}F(\theta) - \bar{\theta}\eta\mu_\sigma\big(\sigma(\theta^*)\big)\frac{\mathrm{d}\sigma(\theta^*)}{\mathrm{d}C} = 0.$$

One also uses the relationship for the region where $\theta \geqslant \theta^*$ that the capital constraint is binding, $C = \alpha\sigma(\theta^*)$. □

A.3. *Proof of Proposition 3*

Let θ^*_{oo} be the threshold level with outside options; θ^*_{no} the threshold level with no outside options, and σ^*_{oo} and σ^*_{no} the corresponding risk limits with and without outside options.

Suppose that $\theta^*_{oo} > \theta^*_{no}$; then comparing (15) with (20), $\sigma^*_{oo} > \sigma^*_{no}$. By concavity of the cash flow return functions using (16), we know that

$$(1-\gamma)\mu_\sigma\big(\sigma^*_{oo}\big)\int_{\theta^*_{oo}}^{\bar{\theta}} \theta\,\mathrm{d}F(\theta) + \gamma\mu_\sigma\big(\sigma^*_{oo}\big)\big(1 - F\big(\theta^*_{oo}\big)\big)\theta^*_{oo}$$
$$< r_D A\big(1 - F\big(\theta^*_{oo}\big)\big) + (r_E - r_D)\alpha.$$

However using (21),

$$r_D A\big(1 - F\big(\theta^*_{oo}\big)\big) + (r_E - r_D)\alpha$$
$$= (1-\gamma)\mu_\sigma\big(\sigma^*_{oo}\big)\int_{\theta^*_{oo}}^{\bar{\theta}} \theta\,\mathrm{d}F(\theta) - \gamma\mu_\sigma\big(\sigma^*_{oo}\big)F\big(\theta^*_{oo}\big)\theta^*_{oo} - (\eta - \gamma)\bar{\theta}\mu_\sigma\big(\sigma^*_{oo}\big).$$

Using these last two expressions would imply that

$$\gamma\mu_\sigma\big(\theta^*_{oo}\big)\theta^*_{oo} + (\eta - \gamma)\bar{\theta}\mu_\sigma\big(\sigma^*_{oo}\big) < 0$$

which is a contradiction. □

A.4. *Proof of Proposition 4*

The first statement of the proposition (Eq. (25)) follows from the definition of $\hat{T}$ and Eq. (26). To derive the optimal ν, substitute into the definition of the objective of the division to get

$$U(\theta) = \gamma\mu\big(\sigma(\theta)\big)\theta - \nu(\theta) - \kappa(\theta)\sigma(\theta).$$

Substituting the representation of utility under incentive compatibility, (13),

$$\underline{U} + \int_{\underline{\theta}}^{\theta} \gamma\mu\big(\sigma(\hat{\theta})\big)\,\mathrm{d}\hat{\theta} = \gamma\mu\big(\sigma(\theta)\big) - \nu(\theta) - \kappa(\theta)\sigma(\theta).$$

Rearranging provides the following sequence of expressions:

$$\begin{aligned}
\nu(\theta) &= \gamma \int_{\underline{\theta}}^{\theta} \hat{\theta}\,\mathrm{d}\mu\big(\sigma(\hat{\theta})\big) - \kappa(\theta)\sigma(\theta) + \gamma\mu\big(\sigma(\underline{\theta})\big)\underline{\theta} - \underline{U} \\
&= \int_{\sigma(\underline{\theta})}^{\sigma(\theta)} \kappa\big(\sigma^{-1}(\sigma')\big)\,\mathrm{d}\sigma' - \kappa(\theta)\sigma(\theta) + \gamma\mu\big(\sigma(\underline{\theta})\big)\underline{\theta} - \underline{U} \\
&= -\kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma\big(\hat{\theta}\big)\,\mathrm{d}\kappa\big(\hat{\theta}\big) + \gamma\mu\big(\sigma(\underline{\theta})\big)\underline{\theta} - \underline{U}.
\end{aligned}$$

Equation (26) is derived from the above equation. □

A.5. *Proof of Proposition 8*

Note that

$$\sigma_p^2 = \sum_{i,j} \sigma_i\sigma_j\beta^2\sigma_m^2 + \sum_j \sigma_j^2\sigma_\epsilon^2.$$

From Eq. (43), we can write

$$\begin{aligned}
\varsigma_i^*/\sigma_i^* &= \alpha\frac{\partial\sigma_p}{\partial\sigma_i} = \alpha\frac{\partial}{\partial\sigma_i}\big[\sigma_p^2\big]^{1/2} = \frac{\alpha}{2\sigma_p}\left[\frac{\partial\sigma_p^2}{\partial\sigma_i}\right] = \frac{\alpha}{\sigma_p}\left[\sum_j \sigma_j\beta^2\sigma_m^2 + \sigma_i\sigma_\epsilon^2\right] \\
&= \frac{\alpha}{\sigma_p}\left[\sigma_i\beta^2\sigma_m^2 + \sum_{j\neq i}\sigma_j\beta^2\sigma_m^2 + \sigma_i\sigma_\epsilon^2\right] = \frac{\alpha}{\sigma_p}\left[\sigma_i + \sum_{j\neq i}\sigma_j\beta^2\sigma_m^2\right].
\end{aligned}$$

Now consider the case that $\sigma_i = k\sigma^*/n$. Substituting this into the expression above gives

$$\varsigma_i^*/\sigma_i^* = \frac{\alpha}{\sigma_p}\left[k\sigma^*/n + k\frac{n-1}{n}\sigma^*\beta^2\sigma_m^2\right].$$

Since

$$\sigma_p^2 = k^2n^2\frac{\sigma^{*2}}{n^2}\beta^2\sigma_m^2 + k^2n\frac{\sigma^{*2}}{n^2}\sigma_\epsilon^2 = k^2\sigma^{*2}\beta^2\sigma_m^2 + k^2\frac{\sigma^{*2}}{n}\sigma_\epsilon^2,$$

we find that in the limit,

$$\sigma_p \to k\sigma^*\beta\sigma_m.$$

Therefore,

$$\varsigma_i^*/\sigma_i^* \to \alpha\left[\frac{k\sigma^*\beta^2\sigma_m^2}{k\sigma^*\beta\sigma_m}\right] = \alpha\beta\sigma_m.$$

By the condition on the parameters (52), we must have that $\beta\sigma_m < 1$ and therefore $\alpha\beta\sigma_m < \alpha$. □

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