Private business returns and the distribution of wealth

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Abstract

This paper develops and calibrates a model of business formation by households and studies the resulting business return and wealth distribution. We interpret the decision to become an entrepreneur as a dynamic portfolio choice problem of a life-cycle investor facing a liquidity constraint and imperfect information about the profitability of potential businesses. In this setting, starting a business is equivalent to investing in non-traded business assets subject to transaction costs. We model the return on those assets as a weighted sum of two components, the individual ability of the business owner and idiosyncratic business risk. Information is imperfect, because only entrepreneurs observe their own business risk realizations. Using numerical techniques we find that the model performs well in matching some key empirical features of the US income and wealth distribution and the structure of asset returns in the US.

Keywords: Wealth distribution, Life-cycle models, Private equity

JEL classification: D31, D91

1 Introduction

In recent years, economists have begun to analyze in depth the determinants of the distribution of wealth and income in modern economies and the relationship between the distribution of wealth and long-run productivity and output growth (see for example Banerjee and Newman (1993), Aghion and Bolton (1997) or Piketty and Saez (2001) and the references therein). A general conclusion of the empirical work in this area, which is reflected in the more theoretical papers, is that the composition of income and wealth varies with total household wealth. Wealthier households tend to have larger business wealth on average and receive a larger fraction of income in the form of capital gains and business profits. At the same time, new datasets have made it possible to study the returns to private business assets at least for the US and the surprising finding of this literature is that expected returns are unexpectedly low. In particular, Moskowitz and Vissing-Jorgensen (2002) find that average ex-post private business returns are comparable to average ex-post returns on stocks. At the same time, the variance of private business returns is much larger than the variance of stock market returns and the return distribution is strongly skewed. They also find that private business asset holdings are quite concentrated with most entrepreneurial households investing the majority of their wealth in a single firm. These two empirical facts, the low expected returns to private business investment and the large share of private wealth held in private businesses seem to be hard to reconcile with each other in a model of rational behavior. Households have a strong incentive to invest in publicly traded assets rather than private businesses if they are risk-averse. The major part of their income would then accrue in the form of dividends and stock market gains rather than business profits. Piketty and Saez (2001) show that this is the case only for the wealthiest 1/1000th of households in the US.

In the following, we propose a model that captures both empirical facts mentioned above, the low expected returns of business assets and the changing composition of household wealth along the wealth distribution. We show that the average realized rate of return to non-traded business assets with an important idiosyncratic risk component and substantial correlation with stochastic labor income is only slightly higher than the return to a riskless, frictionlessly traded asset. Households owning business assets have very concentrated portfolios and derive a substantial part of their income in the form of business profits and capital gains. Total wealth is unequally distributed among the population and private business owners own a large share of total wealth. More specifically, the aggregate value of business assets roughly equals 1/2 of the aggregate value of traded assets, as reported by Moskowitz and Vissing-Jorgensen (2002). Business owning households are on average 3.6 times wealthier than households not holding business assets and 38% of total wealth is owned by business owning households, who represent a share of only 13.6% of the population (these statistics roughly match the estimates provided by Gentry and Hubbard (2000)).

These results emerge from a simple model of occupational choice over the life-cycle. We take the decision to become an entrepreneur as being equivalent to the decision of investing in a private business. While in principle business ownership does not necessarily imply that the business owner also spends his time running the firm, the entrepreneurship literature finds that most private business owners also have an active management interest in the firm they are investing in. Hence, the "single proprietor model" of private owner/manager businesses accounts for most of the data. When giving this interpretation to the choice of becoming a private business owner however, it is also important to consider the special kind of investment environment in which this choice is embedded. First, the choice of becoming a business owner has strong implications for the life-cycle consumption profile. Standard models of portfolio choice need to be augmented to take into account these facts. We argue that the appropriate setting for studying this phenomenon is a life-cycle model with liquidity-constrained, finitely-lived agents who in each period can decide to be business owners or workers. The decision to accumulate assets is endogenous in this framework and motivated by the fact that post-retirement income is considerably lower than wage income. Our model takes lifetime as being stochastic, but stipulates a fixed retirement age. Wage and retirement income is exogenous and stochastic over the life-cycle and we also include a mandatory pension system. Second, the financial market in this setting is inherently incomplete and it is necessary to take into account both, the risk component of starting and running a business, and the transaction costs associated with it. Starting a new business implies creating a new type of financial asset by assembling various capital goods into a legal entity called a firm. This newly created asset is then not traded on an asset market[1]. It is likely that full information about the return characteristics of the asset becomes available only upon creation of the asset and that the transaction costs associated with business ownership are important. In our model, all agents are informed about the conditional distribution of

[1]There is a procedure by which non-traded private equity is converted into traded public equity, which is called Initial Public Offering. We believe that our model is a good starting point for modeling IPO's, but limit our analysis to firms that do not ultimately go public.

the returns to private businesses, but only business owners themselves observe their business risk realization. Due to the incompleteness of financial markets, agents cannot sell short their own business asset or hold claims on other private businesses. It is this short-selling constraint, the existence of transaction costs and the serial correlation in returns to business ownership, which makes the optimal portfolio choice inherently dynamic and requires numerical solution methods.

Wealth heterogeneity emerges from the different income paths and savings incentives for households investing in successful businesses. Due to the serial correlation in private business returns, households owning profitable businesses have superior savings incentives in comparison with other households. These households are characterized by both higher period income from high business returns and higher savings-ratios from higher expected returns to their private business. These households tend to display increasing consumption profiles for at least some time over the life-cycle. The phenomenon of low expected returns and high portfolio share of business assets in a cross-section of households is explained in our model by mainly two factors. First, agents with standard utility functions and standard levels of risk aversion do not require a large excess return to hold asset with calibrated levels of volatility. Second and more importantly, average required rates of return on business assets dependent on the age of the household and the age of the business. Young households require lower average rates of return to start a business than older agents. The persistence and imperfect information about private business returns make it attractive for wealthy young agents to start a business even if the average expected return is low. The reason for this is the possibility of future exit, if the business return realization is bad. Effectively, this means that the present value of the loss associated with low realizations of business returns is bounded. On the other hand, if the business return realization is good, the present value of the gain associated with a successful business can be fully reaped by young agents. Profitable businesses ensure high incomes and rapid wealth accumulation which in turn supports high levels of consumption later in life. Since agents cannot borrow against future income, some highly skilled agents are deterred from starting a business by insufficient wealth. Since the expected rate of return on private equity depends positively on the skills of an agent, especially wealthy individuals with high wage income tend to start private businesses early in life. Later in life, households typically have accumulated some wealth which could be used to start a business. Their future income however is projected to decrease and agents therefore seek to avoid holding a risky asset, even if its returns are relatively high. Together with the fact that the weight of young agents in the

US population is relatively large, these effects lead to relatively low average individual returns in the cross-section. The index return is not affected that much however, because young businesses tend to be small and young agents own young businesses.

The next section provides a brief overview of the literature on occupational choice and optimal portfolios in life-cycle models. Section 3 presents the model in detail and discusses calibration issues. In Section 4 we provide the results for our baseline calibration and analyze the mechanism by presenting results from alternative calibrations. Section 5 concludes and points to interesting research topics in the future. The appendix contains a description of our computational procedure.

2 Related literature

The theoretical literature on private business ownership is unexpectedly small in comparison with the prominence of the topic in the popular press. Following Schumpeter's treatise on economic development and entrepreneurship (Schumpeter (1934)), relatively little academic attention has been devoted to studying this essential phenomenon of capitalist economies. Laffont and Kihlstrom (1979) consider risk aversion as the main determinant of becoming an entrepreneur in a static general equilibrium setting, but fail to provide convincing empirical evidence for their hypothesis. In an interesting and challenging paper, Banerjee and Newman (1993) analyze the effect of liquidity constraints on occupational choice in a dynamic growth model and Aghion and Bolton (1997) present a theory of growth and wealth accumulation Jovanovic (1982) studies the entry and exit decisions of firms in a dynamic model with gradual learning by firm owners about the return structure of their project. Other than Moskowitz and Vissing-Jorgensen (2002), there also exists little empirical work on the return distribution of private businesses. Empirical studies of business ownership have focused instead on finding determinants of the decision to becoming an entrepreneur or being self-employed. Evans and Jovanovic (1989) were among the first to study the determinants of entrepreneurship, focusing in particular on the effect of receiving a large gift or bequest on the probability of becoming an entrepreneur. Subsequent studies such as Holtz-Eakin et al. (1994), Hurst and Lusardi (2002) and Hamilton (2000) enlarge the set of determinants considered, but find similar results concerning the effects of large positive income surprises.

The paper that is probably most closely related to ours is Cagetti and

DeNardi (2002), who study the decision of becoming a private business owner in an overlapping generations setting. These authors focus on the wealth distribution without taking into account the structure of returns to entrepreneurship and while they do employ a general equilibrium setting, their model is also much more stylized. The general equilibrium assumptions require business owners to operate in a secluded sector without hiring labor, allowing business owners to jointly determine the rates of return and the amount of assets accumulated through an entrepreneurial production function. Our model instead takes the distribution of rates of return to entrepreneurship as exogenous, but endogenously determines the share of wealth invested in private equity at each point in time.

A large literature exists on optimal saving decisions over the life-cycle, if liquidity constraints are binding and one asset is available. Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987) were among the first to study this issue. More recent studies including a portfolio-choice component, participation costs and stochastic labor income are Campbell et al. (1999), Haliassos and Michaelides (2002) and Laibson et al. (1998). Campbell et al. (1999) study the implications of investing retirement assets in the stock market rather than government bonds. They find that the slightly higher expected returns of investment in the stock market can be translated into sizeable utility gains by adjusting the age-profile of social security contributions. A combination of lowering social security taxes for young households and investing existing funds partly into the stock market would lead to a lifetime utility increase equivalent to a 3.7 percent increase in lifetime consumption. Laibson et al. (1998) build a detailed model of the US economy taking into account heterogeneity in wealth and education levels, but focus on the optimal saving behavior of consumers with time-inconsistent preferences and its implications for public pension schemes.

3 Modeling business formation

Like the portfolio choice models of Campbell et al. (1999) and Laibson et al. (1998) our life-cycle problem is set in a partial equilibrium context, taking the stochastic processes of asset returns and wages as given. Individuals have finite, stochastic lifetimes and we set a fixed retirement date. There is no bequest motive.

Household $i \in \{1, 2, ..., I\}$ maximizes utility over a finite horizon of T

periods:

$$\max_{c_{it}, B_{i,t+1}, S_{i,t+1}} E_0 \left[\sum_{t=1}^{T} \delta^t \left(\prod_{j=1}^{t} p_j \right) u(c_{it}) \right], \tag{1}$$

where δ is a discount factor and p_t denotes the probability of being alive at date t, conditional on being alive at date $t-1$, subject to the following budget constraint

$$\begin{aligned} c_{it} = w_{it} + (1+r)B_{it} - B_{i,t+1} - \theta w_{it} + (1+v_{it})S_{it} - S_{i,t+1} - ... \\ ... - \mathbf{1}(S_{it} = 0, S_{i,t+1} > 0)\Phi - \mu \max(0, S_{it} - S_{i,t+1}) \end{aligned} \tag{2}$$

and short-sale constraints on the two available assets

$$B_{it} \geq 0, S_{it} \geq 0, \forall t \tag{3}$$

Exogenous income has an age-dependent deterministic component

$$\bar{w}_t = g(t), \text{ for } 1 \leq t \leq P$$

and

$$\bar{w}_t = \bar{b}, \text{ for } P < t \leq T$$

where P denotes the fixed retirement age. In addition, there is uninsurable idiosyncratic income risk ε_{it} such that

$$\log w_{it} = \log \bar{w}_t + \varepsilon_{it}$$

The stochastic income component is described by

$$\varepsilon_{it} = \phi \varepsilon_{i,t-1} + \upsilon_{it},$$

where υ_{it} is an iid shock distributed as

$$\upsilon_{it} \sim N\left(0, \sigma_\upsilon^2\right).$$

There are two types of assets available in the economy, a financial asset traded in public markets and denoted by B_{it}, which yields a certain rate of return r. The other type of asset is provided by capital invested in private businesses managed by the household. This non-traded business asset yields an idiosyncratic return

$$v_{it} = \bar{v} + \rho \varepsilon_{it} + \zeta_{it}, \tag{4}$$

which consists of an average guaranteed return, $\bar{v}$, a skill component, perfectly correlated to the uninsurable idiosyncratic income risk, $\rho\varepsilon_{it}$, and a business risk component, ζ_{it}. The business risk component is orthogonal to the skills component and evolves according to

$$\zeta_{it} = \psi\zeta_{i,t-1} + \xi_{it},$$

where ξ_{it} is an iid shock distributed as

$$\xi_{it} \sim N\left(0, \sigma_{\xi}^{2}\right).$$

and every newly created business receives an initial draw from the steady state distribution of ζ.

A crucial feature of the model are the assumptions relating to the information on the uncertainty realizations available to the agents. We assume that each individual can observe the realization of idiosyncratic income risk at each point in time. However, it is assumed that business risk is unobserved prior to starting a business. This implies that only business owners, defined as having a portfolio with a strictly positive level of private business assets, can use conditional distributions of returns in their optimal decisions by applying transition probabilities over the states of business risk. Non-entrepreneurs are facing the unconditional distribution of business risk.

Investment in a private business is subject to transaction costs, which represent startup costs, $\mathbf{1}(S_{it} = 0, S_{it+1} > 0)\Phi$, and partial investment irreversibility, $\mu \max(0, S_{it} - S_{it+1})$. As in Abel and Eberly (1994) or Dixit and Pindyck (1998) μ represents the wedge between the purchase and sale price of private business assets[2].

The structure of this problem is close to a standard dynamic portfolio choice problem for a finite horizon investor introduced by Samuelson (1969) and Merton (1969). The distinguishing features of our model are the existence of transaction costs, the fact that returns to private businesses are correlated with exogenous income, and the informational assumption that returns to the business asset are observed only when its share in the portfolio is strictly positive. These assumptions together with the short sale constraints require the use of numerical techniques to find the solution of the problem.

[2] The idea here is that selling private equity essentially means selling some of the capital goods used in the business. The wedge therefore implicitly exists between the purchase and sale price of capital goods.

4 Calibration

We set the maximum lifetime of agents in our model to 89 years and assume that agents enter the model at age 20. In order to economize on computational resources, we choose a period-length of 3 years and hence arrive at a total lifetime of 23 periods. The mandatory retirement date is the end of period 16, which is equivalent to an age of 68 years. Death probabilities are taken from the lifetables of the US National Center for Health Statistics, which report conditional survival probabilities[3].

Agents are assumed to maximize a standard instantaneous utility function of the CRRA-class,

$$u(c_{it}) = \frac{c_{it}^{1-\sigma} - 1}{1-\sigma}$$

setting the coefficient of relative risk aversion equal to 0.5 and the discount rate to 0.97. These values are common in the literature (see Conesa and Krueger (1999), Campbell et al. (1999) and Cagetti and DeNardi (2002)) and consistent with estimates by Gourinchas and Parker (1999) in their analysis of lifetime consumption profiles.

Fixed Parameters	Value
σ	2
δ	0.97
r	0.05
θ	0.185
ϕ	0.688
σ_v^2	0.052

We take the calibration of the wage income process from the detailed study by Laibson et al. (1998). These authors estimate age-income profiles for three educational groups from PSID data by regressing log income on powers of age and some control variables. We select the median educational group as being most representative for the average household.

$$g(t) = \exp(8.835 + 0.058t - 0.017t^2/100 - 0.055t^3/10000)$$

We convert their results obtained for annual data to a 3 year-frequency by time-aggregation. The autocorrelation coefficient of the stochastic wage

[3] U.S. National Center for Health Statistics (1994)

process is equal to 0.688 for annual data, which implies a coefficient of 0.326 for our calibration. The variance of the innovation to log income is 0.052 for annual data, resulting in a value of 0.0883 for our modeling frequency.

We approximate the tax system by a proportional tax on exogenous income, choosing a tax rate of 18.5% and calibrate the deterministic component of retirement income to match the average replacement ratio[4] of 45% reported in Laibson et al. (1998) and Engen et al. (1994).

Draws of initial wealth are taken from a lognormal distribution, using the empirical mean of the wealth distribution for the youngest cohort (equal to \$23,183) and the corresponding coefficient of variation of 6.53 given in Budría et al. (2002).

In the aggregation we used population weights from the 1998 issue of the CPS (Current Population Survey), truncated below age 20, assuming a long-run real income growth rate of 1%.

Calibrated Parameters	Value
ψ	0.867
σ_{ξ}^{2}	0.004
$\bar{v}$	0.06
Φ	1.835
μ	0.1
ρ	0.4

We now discuss the calibration of the asset returns and transaction costs. For obvious computational reasons, we have restricted the type of assets in our model to two. The traded asset is supposed to capture the investment possibilities in the public equity and bond market. The return of the traded asset is assumed to be fixed to further reduce computational costs. The annual rate of return on the publicly traded asset is assumed to be 5%, in line with estimates of long-run average returns to public equity and bonds reported in the finance literature (see for example Mehra and Prescott (1985) or Heaton and Lucas (1997)). The deterministic component of the private business asset return $\bar{v}$, equal to the unconditional mean of the return distribution, is chosen to imply an empirically plausible share of entrepreneurs in the population. Gentry and Hubbard (2000) consider various definitions of entrepreneurship and report shares of 8.7% to 11.5% in the population

[4] Defined as the ratio of pension benefits to wage income in last working period.

with 8.7% being their preferred interpretation. The total return to private businesses also includes a skill and a business risk component. As pointed out above, the skill component is equal to the stochastic component of the income process. We choose the autocorrelation coefficient ψ, the weight of skills in the return distribution and the variance of the innovation to business risk σ_ξ^2 in order to achieve a relatively wide distribution of possible asset returns. Our baseline calibration sets ψ equal to 0.867, ρ equal to 0.4 and σ_ξ^2 equal to 0.11. These values translate into a span of -56.31% to 94.52% for the unconditional distribution of private business returns in the calculation using a grid that is 6 standard deviations wide.

Transaction costs are chosen to be of a plausible magnitude compared to wages. We set the startup cost parameter Φ equal to 1.835, equivalent to about one annual income for a young household. This value is subject to some uncertainty, but our choice seems to be realistic and relatively prudent as Caballero and Hammour (1994) used a value equal to half a year of production costs for manufacturing in their calibration. Costly reversibility is captured by the parameter μ, which we take to be relatively low with a value of 0.1. Also for this parameter estimates in the literature vary widely. Dixit and Pindyck (1998) for example assumed a wedge of 40% in their illustration of the importance of costly reversibility for the behavior of firm investment. Obviously, the calibration of the parameters pertaining to business risk and transaction costs is the most difficult, because there are not studies much in the literature and therefore little empirical evidence is available. For this reason, we will provide some sensitivity analyses for the values of these parameters in the following section.

5 Private business returns

Our model matches quite well the relative sizes of the private corporate business sector and the size of public equity and bond markets. Moskowitz and Vissing-Jorgensen (2002) find that the size of the US private corporate business sector roughly equals the size of the US public equity market, which in turn roughly equals the size of the US domestic securities market. Our model also implies a size ratio of 2 : 1 for the publicly traded asset market to the non-traded private business sector.

We follow Moskowitz and Vissing-Jorgensen (2002) to calculate private

Figure 1: The cross-sectional distribution of private business returns

equity returns as

$$v_{it} + \left(\frac{\text{Value at the time of the survey}}{\text{Value of original investment}} \right)^{1/(\text{Years since founded})} - 1$$

where "Value at the time of the survey" takes into account costly reversibility and "Value of original investment" includes startup costs.The return to an index of all private businesses is calculated as 7.13% annually (22.9% for a 3 year horizon), resulting in an excess return of a private business portfolio of 2.13%, far lower than the 10% calculated by Heaton and Lucas (1997). Even more interesting, the cross-sectional average return (even conditional on survival) is much lower than the index return and equals 5.03% annually (16.0% for a 3 year horizon), almost equivalent to the rate of return on the riskless asset. At the same time, Figure 1 shows that the cross-sectional ex post-return distribution is very wide, ranging from −64.06% to +85.2% for a 3 year horizon, with a standard deviation of 0.18. Hence, although the average business owner does not earn a rate of return above the public market rate, and investing into a private business is associated with substantial idiosyncratic risk, households are still willing to take the risk of business ownership.

In fact, our baseline calibration generates a share of private business owners in the population that is slightly higher than the one reported by Gentry

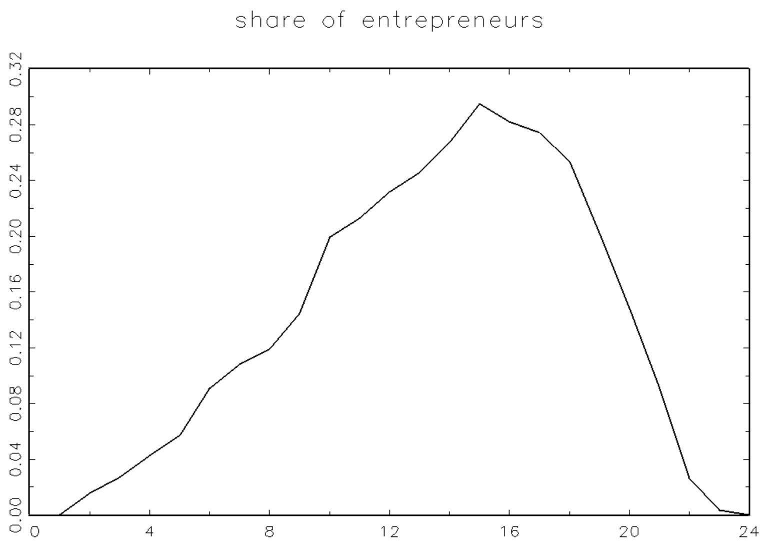

Figure 2: Share of business owners by age

and Hubbard (2000) with 13.8%. This suggests that average returns could even be slightly lower in order to match US empirical data. Figure 2 also shows that just as in the data, the share of entrepreneurs in the population is hump-shaped with respect to age. Bregger (1996) reports that the ratio of the number of self-employed workers to total employment is increasing with age. Towards the end of the life-cycle, agents tend to sell their businesses and keep their wealth in publicly traded assets.

Although average returns are low and the volatility of returns is high, business owning households hold large shares of their wealth in private equity of a single firm. Our model predicts an average of 88.7%, which is quite close to the 82% reported by Moskowitz and Vissing-Jorgensen (2002). While most agents do not hold private business assets at all, those that do have large holdings relative to their own wealth. Median firm age is less than average firm age, indicating that most firms have relatively short lives. In fact, the survival rate of firms above 12 years is only 36.1%, approximately equal to the 1/3 survival rate for a 10 year horizon quoted in the literature.

The determinants of becoming an entrepreneur are also quite appealing. In our model, entrepreneurs tend to be more skilled and have more initial wealth than the population average. A very interesting aspect of our model

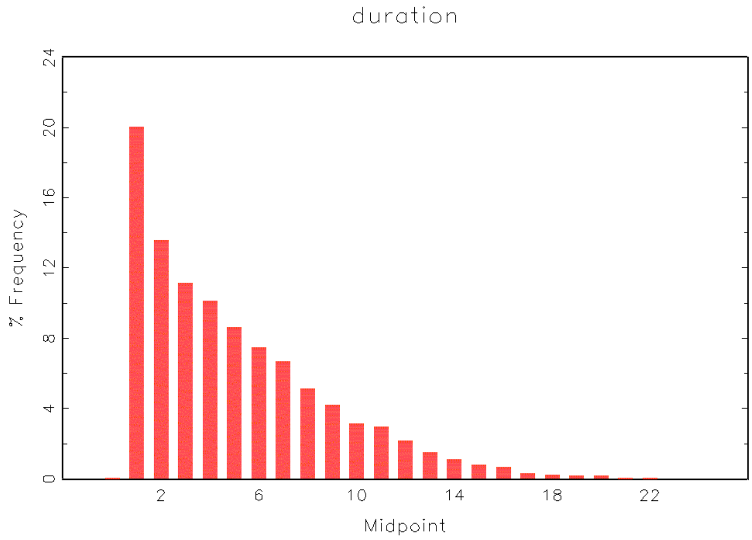

Figure 3: Histogram of firm age

is the fact that average rates of return to private equity vary systematically with the age of entrepreneurs and the age of the firm. Young households require low average rates of return to start a business and young households own young firms.

Therefore, also young firms have relatively low rates of return. Because households exercise the option to exit if initial returns are too low young firms also have relatively large failure rates. Average firm age in our baseline calibration is 12.6 years, only slightly higher than the 10.7 years reported by Moskowitz and Vissing-Jorgensen (2002).

6 The income and wealth distribution

Household income and wealth heterogeneity in our model is created by idiosyncratic risk that affects both labour income and private business returns. This risk should be interpreted as representing any form of idiosyncratic difference between agents as well as pure chance. We do not make any attempt to separate these two causes of income differences. The income and wealth distribution of the US economy has recently been studied intensively

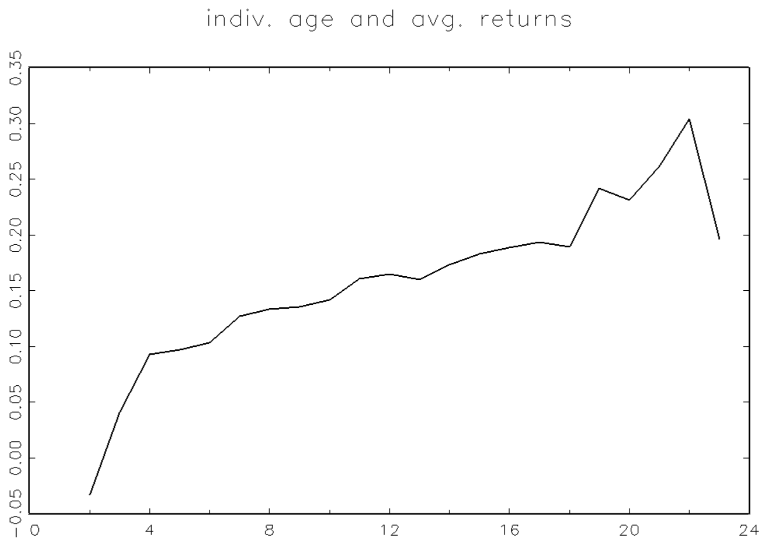

Figure 4: Average private business returns by household age

Figure 5: Average private business returns by firm age

by macroeconomists interested in the long-run evolution of wealth and income inequality and its relationship to long-run economic growth. Piketty and Saez (2001) document the evolution of the top decile of the US income distribution in the 20th century using individual tax filings. Budría et al. (2002) report the US earnings and wealth distribution emerging from the 1998 Survey of Consumer Finances.

The most salient features of the US income and wealth distribution are that both are skewed with the median smaller than the average and that in the cross-section income and wealth are positively correlated. Budría et al. (2002) report a correlation coefficient of 0.60 between income and wealth and Gini coefficients of 0.55 for the income distribution and 0.80 for the wealth distribution. Interestingly, over the life-cycle earnings inequality increases while wealth inequality decreases. For young households a relatively low Gini coefficient for income of 0.43 is coupled with a very high Gini coefficient for wealth of 0.91. For households later in the life-cycle the Gini coefficient for income increases to 0.67 and the Gini coefficient for wealth decreases to 0.73. In the baseline calibration, the model does a quite good job of matching these statistics. The correlation between income and wealth in our simulated economy is 0.52. The Gini coefficients for income and wealth are 0.25 and 0.57 which shows that endogenously generated wealth heterogeneity is much larger than the largely exogenous income heterogeneity as it is in the data. Allowing for more income heterogeneity than is implied by our estimated income process will further increase inequality. The evolution of income and wealth inequality is in line with the empirical evidence. Wealth heterogeneity of households in young age (model age 2) implies a Gini coefficient of 0.83, similar to the data and this coefficient decreases to 0.33 over a period of 40 years (model age 15). Income heterogeneity in contrast increases from 0.18 to 0.25 for those ages.

Also concerning the wealth distribution by occupation our model fits the empirical data quite well. The wealth distribution is strongly skewed to the left with median wealth lower than average wealth for any age group and both entrepreneurs and non-entrepreneurs. The share of total wealth held by entrepreneurs is 38.1%, which corresponds closely to the 37.7% reported by Gentry and Hubbard (2000). As in US data, entrepreneurs are also significantly wealthier on average than non-entrepreneurs. In our baseline calibration, business owners hold on average 3.9 times the average wealth of workers, while Gentry and Hubbard (2000) report an average wealth ratio of 6.8 (Budría et al. (2002) report 5.6 for the average wealth ratio between workers and business owners).

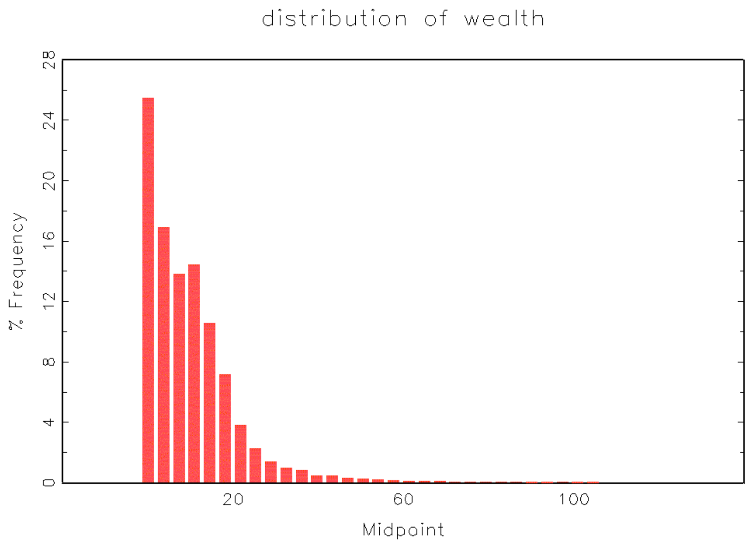

Figure 6: The cross-sectional distribution of wealth

With respect to income, Budría et al. (2002) find a ratio of 2.2 for the average income of business owners to the average income of worker households and in the baseline calibration we obtain a similar value of 1.74 . Both Piketty and Saez (2001) and Budría et al. (2002) report systematic variation of the importance of various sources of income with household income and wealth. Generally, higher wealth or income implies a larger share of capital income and business profits in total income. Ranking households according to their wealth, the first quintile receives 98.2% of total income from wages and transfers according to empirical data. Our model economy implies a fraction of 100%. The third quintile of the wealth distribution receives a somewhat smaller share of income from wages and transfers (94.6% empirically and 87.6% in our baseline model). The composition effect is most pronounced for the highest wealth quintile. The richest 20% of households receive only 59.2% of total income from wages and transfers according to SCF data reported in Budría et al. (2002). 21.6% of total income is provided by capital income and 17.6% stem from private business profits. Our model economy again implies similar shares of 51.3%, 17.2% and 31.4%, respectively. Business profits and capital income become even more important for higher fractions of the wealth distribution accounting for 65.5% of total income in the highest percentile empirically (77.5% in our model economy). Figure 6 below displays a graph of the cross-sectional household wealth distribution in our baseline calibration.

The differences in the composition of wealth and income between private business owners and other households also lead to differences in consumption patterns. In general, the consumption profile of private business owners is different than the consumption profile of the general population, with private business owners having increasing rather than hump-shaped consumption profiles. This behavior is consistent with different savings incentives for these households, a hypothesis formulated by Quadrini (1999) in his study of wealth concentration, social mobility and entrepreneurship. The following section is dedicated to a detailed analysis of the sensitivity of these results to changes in the calibration of the model and an exploration of the mechanism generating the results presented above.

7 Sensitivity analysis

In the following, we take the set of fixed parameters and the deterministic component of private business returns $\bar{v}$ as given and concentrate on the remaining calibrated parameters. Our results are summarized in Table 1, which gives summary statistics for the average, median and standard deviation of cross-sectional returns on private business assets, the average share of total wealth held in private business assets conditional on running a private business and the Gini coefficients for the income and wealth distribution in our simulated economy.

Table 1:

	Cross-section of private business returns			Mean private business	Gini coefficients
	Excess mean return	Median return	Stand. Dev.	portfolio share	Wealth/Income
BASELINE	0.23%	16.32%	0.182	0.88	0.57/0.25
FIXEDCOSTHIGH	0.27%	17.07%	0.198	0.83	0.55/0.24
DISINVESTHIGH	−2.37%	13.51%	0.186	0.76	0.58/0.27
PERSISTENCELOW	−1.14%	15.65%	0.181	0.80	0.57/0.25
OBSERVABLE[1]	0.10%	16.14%	0.181	0.81	0.59/0.27
LIFECYCLE	5.99%	19.10%	0.175	0.94	0.57/0.28

[1] For this run, all agents are allowed to observe their business risk realization

The first column contains the name of the parameter set used. The actual parameter values used for each calibration can be read from Table 2 below.

Table 2:

	ψ	σ_ξ^2	Φ	μ	ρ
BASELINE	0.867	0.004	1.835	0.1	0.4
FIXEDCOSTHIGH	0.867	0.004	3.670	0	0.4
DISINVESTHIGH	0.867	0.004	0	0.25	0.4
PERSISTENCELOW	0.546	0.011	1.835	0.1	0.4
OBSERVABLE[1]	0.867	0.004	1.835	0.1	0.4
LIFECYCLE	0	0.016	0	0	0.4

[1] For this run, all agents are allowed to observe their business risk realization

7.1 Alternative transaction cost structures

Varying the structure of transaction costs does not have a large effect on our main results. Neither the presence of fixed startup costs, nor the presence of costly reversibility are necessary to reproduce our main findings. With respect to other interesting statistics, disregarding either of these costs worsens the performance of our model however. Neglecting startup costs has the effect of changing the age structure of private business owners and the size distribution of firms. Most business owners start their business very early in life and some keep relatively small businesses. This behavior is clearly related to the assumption of non-observable and persistent business risk, implying that is worth running a small, but not highly profitable business, in order to be better informed about business return realizations. Figure 7 illustrates this by plotting the size of distribution of firms for agents in period 14.

The share of business owners also rises very quickly and is too high for this alternative set of parameters.

Neglecting costly reversibility and setting startup costs twice as high as before ($36,900) leads to startups later in life. The average age of entrepreneurs rises to 60.8 years, from an already relatively high number of 56.6 years in the baseline run. Moskowitz and Vissing-Jorgensen (2002) report an average age of entrepreneurs of 46.5 years, closer to the 50.6 years resulting from the case without startup costs, but high disinvestment costs. Also, the increased startup costs lead to a bimodal distribution of portfolio shares (Figure 8), with some agents avoiding to exit the business completely, but allocating only a small share of their wealth to their private business in some periods, when returns are comparatively low.

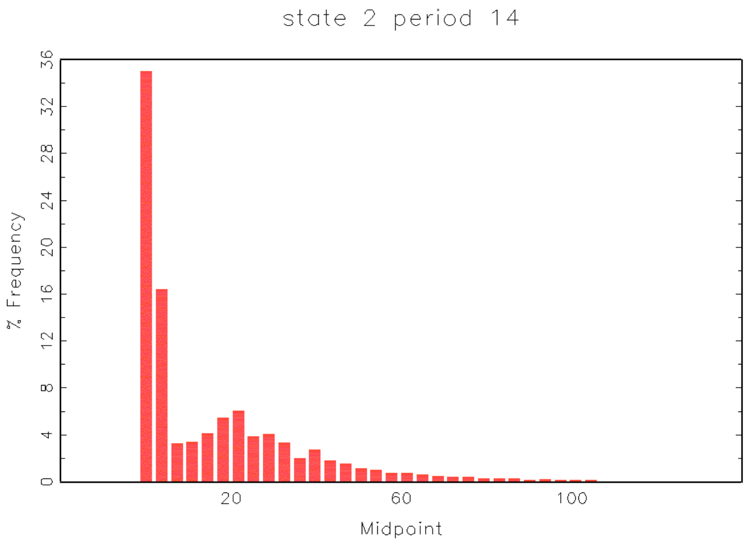

Figure 7: Firm size distribution in period 14

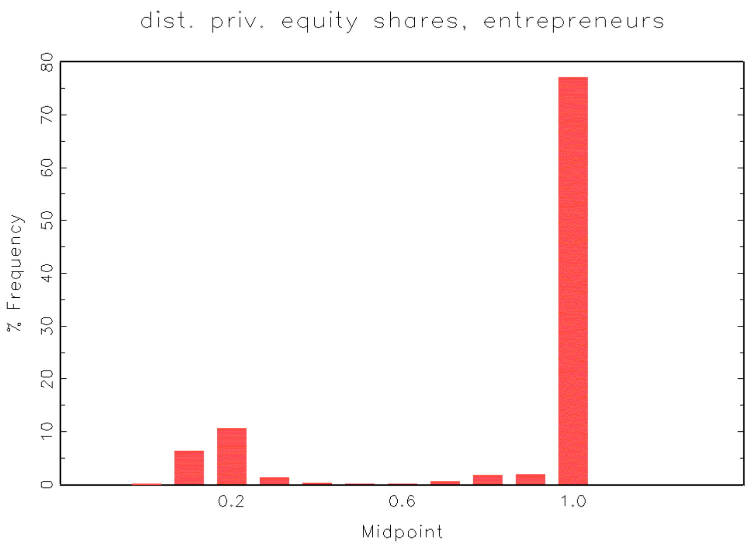

Figure 8: Share of wealth invested in a private business

7.2 Persistence in business risk

The parameters defining the distribution of private business returns are hard to calibrate, but at the same time fundamental for the question we would like to answer. Both, the persistence of business risk realizations and the volatility of the private business returns play an important role in the model. Concerning the persistence of business risk realizations, there is no information to be gained from the SCF, since it is not a panel study and few alternative information sources exist. Studies (e.g. Quadrini (1999)) on the wealth distribution and on the savings choices of entrepreneurs suggest that the persistence in total returns to private business assets is relatively high. In our model, the total return is the sum of two components, a skill component which has some persistence (the first autocorrelation coefficient is approx. 0.4 and the autocorrelation function declines exponentially) and the business risk component. In order to check the robustness of our results with respect to the persistence in business risk, we have also computed a model with little persistence in business risk, such that the main source of serial correlation in private business returns is the skill component. Results are robust in the sense that our main findings do not change even if business risk is less persistent. In fact, average private business returns are even lower than in the baseline scenario because entrepreneurs do not exit as quickly after observing a bad business risk realization. However, big changes occur in a different dimension. The average lifetime of private businesses increases drastically if business risk is less persistent and the shape of the survival rate distribution changes. Figure 9 shows the histogram of firms' lifetimes for the case of low persistence in business risk, labeled PERSISTENCELOW. Very few businesses live for less than twelve years, and the average is 10.16 periods, corresponding to approximately 30 years in real time. This is much shorter than estimates in the literature reported above. Therefore, we consider our baseline specification assuming higher persistence as the more appropriate one.

7.3 Observability and pure life-cycle effects

We next turn to the effects of perfect information on the part of potential entrepreneurs about their current business risk realization. With observability the risk associated with starting a private business is sharply reduced. Almost every household runs a private business at some point in time and the average duration of firms is high. Figure 10 plots the histogram of startups per agent for our population. The reason underlying this behavior is that

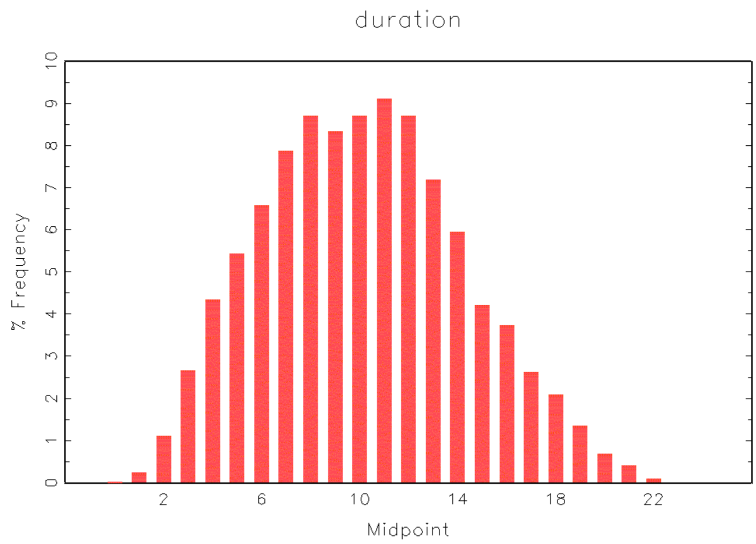

Figure 9: Age distribution of firms

agents observing their draws of business risk are able to "time" their start-ups. They enter when their business risk realization is high and they remain in business as long as returns remain high. The average duration of private businesses is higher than in the baseline case, because there are no exits due to households learning about low business return realizations. There is only a small premium required for running a private business because agents are relatively well-informed about future returns, once they observe their current business risk realization. The wealth distribution is the most unequal among all the calibrations studied, because households can fully exploit positive realizations of business risk.

To obtain a comparison with standard models, we have also examined the private business returns and the wealth distribution that would emerge from a canonical model of portfolio choice over the life-cycle, eliminating transaction costs and persistence in business risk. Information about the current realization of business risk now does not help in forecasting future business risk realizations and therefore also the informational assumption is not relevant. In such a stripped down model, the only reason for observing results different from standard static portfolio choice models is the existence of the liquidity constraint and the fact that wage and private business returns are highly correlated.

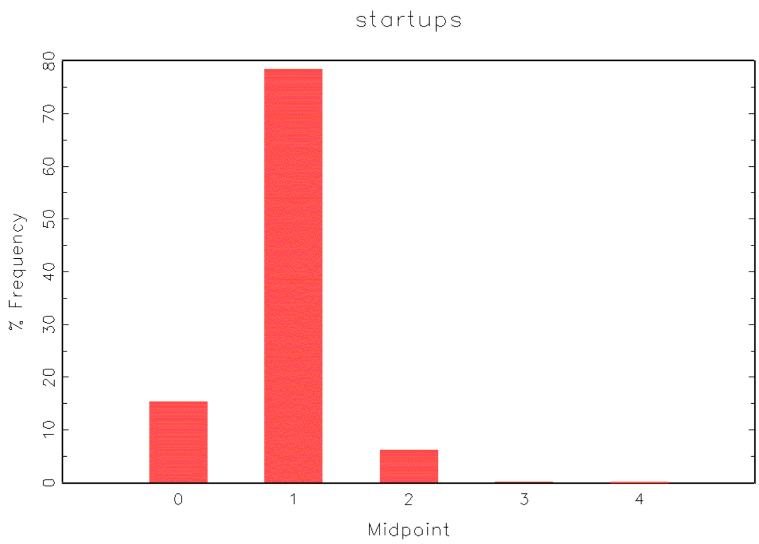

Figure 10: Number of startups per household

As expected, the index and cross-sectional average return to private business assets are basically equal. Private business assets are still very concentrated in household's portfolios due to the skill component in returns, which makes returns conditional on a good skill realization very attractive in comparison with the riskless asset. Information about business risk is irrelevant for the allocation of household wealth, because it is iid. The average number of startups per household is extremely high with 3.1 startups per household. Due to the absence of both persistence in business risk and transaction costs, households frequently invest their wealth into new private businesses. The underlying reason for this is the correlation of business returns with skill realizations. As expected returns change, households shift their private wealth into and out of the private business sector. Frequent shifting is not burdened by transaction costs and hence households extensively make use of this flexibility. Conditional on investing in business wealth, households choose an extreme concentration of their private wealth The high correlation of business returns with wage income induces households to require a sizeable premium of 5.99% per period however. As should be expected from the analysis of static models of portfolio optimization in the absence of transaction costs and persistence, expected returns do not depend strongly on the household's age. The working of the liquidity constraint in the initial period of life leads to a slightly downward sloping age profile of mean cross-sectional returns. Only when households have accumulated some assets and the liquidity constraint is no longer binding expected returns are flat. The wealth and income distribution results obtained in the baseline calibration by and large also carry over to the pure-life cycle case. The distribution of wealth is more unequal than the distribution of income and over the life-cycle Gini coefficients increase for income while they decrease for wealth. Also the composition if household income varies with household wealth in the same way as in the baseline case. Because there are more startups and business-owning households, worker and entrepreneur households are more similar to each other however. The average income of entrepreneurs is only 1.48 times the average income of workers and even low wealth households invest significant fractions of private wealth into private business assets.

8 Conclusions

We have shown in this paper that a simple model of occupational choice in a life-cycle framework can account for some of the key findings concerning private business ownership and the wealth and income distribution in the US. Our main findings are that in a cross-section of households expected returns on private business assets are approximately equal to expected returns on a publicly traded asset. At the same time, the cross-sectional distribution of private business returns is much more variable than the distribution of public asset returns. Households investing in a private business also invest a large share of their wealth to their private business and cross-sectionally, business owner households have a much larger share of income and wealth than non-business owning households. Both the Gini coefficients of wealth and income and the composition of total household income vary over the life-cycle in the same way as in empirical data. Younger households display a higher variation of wealth that decreasing with age and a lower variation of income increasing with age. Wealthier households derive a larger fraction of income from capital income and business profits. Our sensitivity analysis shows that these findings are quite robust to different specifications of transaction costs. However, the existence of transaction costs and the non-observability and high persistence in business risk realizations are necessary ingredients to generate these results.

The model takes a partial equilibrium perspective and studies the endogenous choice of wealth accumulation and portfolio allocation of households that can decide each period on whether to invest in a privately held business. Households in this model also do not take a "dynastic" perspective and any bequests generated within the model are purely accidental. Extending the model to an infinite horizon, general equilibrium setting would be useful to check the robustness of our results to an endogenously generated, stationary wealth distribution. The main difficulty with this approach is the problem of finding a suitable specification of the function generating private business returns. While little is known about realized private business returns, even less is known about potentially realizable private business returns. An extension of the model in this direction would however clearly be useful.

We believe that our model would ultimately also be able to yield important insights into the determinants of business ownership that could prove to be useful for economic policy. To avoid further complications of the model we have largely neglected tax-issues so far, but intend to study the impact of tax and social security systems on the propensity of private business ownership in future work.

9 Appendix

9.1 Numerical procedure

The numerical method we use is finite state, finite horizon dynamic programming. We discretize the state space, defined over asset stocks and uncertainty states. Fineness of the grid basically determines computation time, which is linear in lifetime and quadratic for each asset in the number of gridpoints considered.

Given some discretization of the state space, for each point in time $t = 1, 2, ..., T$, optimal policy rules are computed, describing the optimal adjustments in the levels of the two assets in the portfolio between t and $t + 1$. These rules are made given the level of each asset when taking a decision and conditional on the states of uncertainty over idiosyncratic income risk and business risk observed prior to the decision. Optimal decisions are recursively computed, starting by setting the level of assets in $T + 1$ to zero, and then updating the continuation value at each point in time, according to the optimal decisions between the final period and the current period. While those optimal policies are found by going backwards from the end of life to the beginning of life, the actual solution paths for each individual $i \in \{1, 2, ..., I\}$ are found by applying the optimal policy rules going forward from the beginning of life to the end of life. At the very beginning of life, individuals differ only according to the amount of initial wealth they are endowed with. During lifetime they differ according to the their individual histories of idiosyncratic income shocks and business risk realizations. This explains the distribution of solution paths over assets and consumption in the population of I[5] individuals.

Having obtained the solution paths for the entire population according to individual draws of initial wealth and shock histories, it is straightforward to aggregate and to calculate statistics for the population. The basic problem is to translate a set of longitudinal observations for a population of individuals into a representative cross-section at a given point in time. First, we make sure that the age-structure in the observed population (as taken from the CPS) corresponds to the age-structure of the simulated economy. Therefore, starting from the longitudinal simulated paths for the I individuals, we split the sample into T age groups, such that for each age the its share in the population corresponds to its share in the CPS. Second, the simulated values for each age group are translated into a common unit of account, by adjusting for the aggregate real income growth rate.

Aggregation and the number of individuals do not increase computation time by much, once the optimal decision rules are computed, the marginal computational cost of further numerical analysis is low.

The dynamic programming techniques used in this paper require a special structure due to the informational assumptions about returns to private businesses. For computing optimal decision rules we must distinguish two groups of agents. The first group are the business owners who can observe business risk and therefore compute optimal decisions conditional on the observed realizations of uncertainty in both dimensions, business and idiosyncratic income risk denoted by ζ_{it} and ε_{it}, respectively. The second group are the non-business owning households for whom policy rules are computed conditional only on the observed realizations of idiosyncratic income shocks and using the unconditional distribution of business risk. When simulating the optimal paths for the individuals those two sets of rules must be applied appropriately, depending on whether at the point of decision the individual invests into a private business or not.

[5] We simulate a population of $I = 50000$ individuals to compute our statistics.

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